Correspondence

Effects of Range-Doppler Coupling on Chirp Radar Tracking Accuracy

Abstract

Attention is called to the phenomenon of "range-Doppler coupling," characteristic of linear FM (chirp) waveforms, and the strong effect this coupling may have on tracking accuracy. Numerical results are presented for simple three-state filters and for a realistic reentry vehicle tracking problem.

I. Introduction

The linear FM, or "chirp", waveform [1], [2] is widely used in both radar and sonar because it combines the advantages of pulse compression with great ease of implementation. One of the peculiarities of such waveforms is their strong "range-Doppler coupling" property, the significance of which is often not adequately appreciated. Our intention here is to demonstrate, through numerical examples, the strong effect this coupling can have on tracking accuracy, and the important role played by the *sign* of the frequency sweep.

The linear FM pulse is characterized by a constant rate of change of frequency from f_1 to f_2 over the duration T of the pulse. The filter which compresses the return pulse must therefore introduce a time delay τ which is a linear function of frequency, of appropriate slope:

$$\tau = \tau_0 - [T/(f_2 - f_1)]f. \tag{1}$$

The effect of this characteristic is that the earliest portion of the return pulse is delayed most, in such a way that the output from the compression filter is a very narrow pulse. It is evident from (1) that any Doppler shift Δf in the return pulse, due to nonzero range rate $\dot{r}$.

$$\Delta f = -[(2f_0)/c]\dot{r}, \tag{2}$$

will introduce an increment in the time delay

$$\Delta\tau = [(f_0 T)/(f_2 - f_1)]\,[(2\dot{r})/c], \tag{3}$$

which is indistinguishable from the effect of a change in the range r. Thus, conversion of the measured time delay to range units (by multiplication by $c/2$) does not yield a measurement of range alone, but of the quantity

$$m = r + \dot{r}\Delta t \tag{4}$$

where the quantity

$$\Delta t = (f_0 T)/(f_2 - f_1) = (f_0 T)/B \tag{5}$$

is a characteristic of the waveform alone, and not of the

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target motions. It depends only on the pulse duration T, the center frequency f_0, and the swept bandwidth

$$B = f_2 - f_1, \tag{6}$$

which, it should be noted, is positive for a frequency upsweep $f_2 > f_1$ and negative for a downsweep.

It is evident from (5) that Δt may have a magnitude many times greater than the pulse duration. For most typical radar pulses it is still only a few milliseconds, but in some cases it may reach magnitudes of a second or more, and in sonar applications it may be measured in minutes. As an example, a 200 μs radar pulse at a carrier frequency of 5000 MHz (C band), with a swept bandwidth of 1 MHz, will yield

$$\begin{aligned}\Delta t &= [(5000 \times 10^6)(200 \times 10^{-6})/(1 \times 10^6)] \\ &= 1 \textit{ second}.\end{aligned} \tag{7}$$

The form of (4) has important implications with respect to resolution of multiple targets. Two targets can be more easily distinguished if they present a large magnitude of the measurement difference

$$\begin{aligned}\Delta m &= m_1 - m_2 \\ &= (r_1 - r_2) + (\dot{r}_1 - \dot{r}_2)\Delta t \\ &= \Delta r + \Delta \dot{r} \Delta t.\end{aligned} \tag{8}$$

This implies that a positive Δt is helpful if Δr and $\Delta \dot{r}$ have the same sign (i.e., if the magnitude of the range difference is increasing), which typically occurs when two objects are following the same path at different speeds, with the faster object leading the slower one. This situation exists, for example, in the case of a reentry vehicle and its following wake, or in the launching of a missile from a slower-moving launch vehicle.

On the other hand, a negative Δt will be helpful for resolution if the range difference is decreasing. Such cases are important, for example, in air traffic control (collision avoidance) and in command-guided intercept problems.

II. Simple Tracking Filters

A consideration separate from the resolution question is that of tracking accuracy, which is the aspect to be discussed here. The form of (4) dictates certain modifications to the tracking algorithms, and can have a surprisingly large effect on tracking accuracy.

One approach to the tracking problem is simply to treat the measurement m as though it were a measurement of range alone, and pass it through a simple tracking filter of the "*ghk*" or "$\alpha\beta\gamma$" type [3], [4]. This yields estimates of the quantity $r + \dot{r}\Delta t$ and its first two derivatives. Since $r + \dot{r}\Delta t$, rather than r, is the quantity which should be used for setting the range gates, this approach may be adequate for some applications. In general, however, it is preferable to have separate estimates of r, $\dot{r}$, and $\ddot{r}$. This is particularly important when the radar waveform (and hence Δt) changes, when the target is handed over from one radar to another, or when accurate prediction is required, as for interception of reentry vehicles.

Direct filtering of $r + \dot{r}\Delta t$, as described above, also leads to difficulties in the construction of an optimal filter and in the derivation of the r, $\dot{r}$, and $\ddot{r}$ estimates. If, however, we retain

$$\mathbf{x} = [r\, r\, r\,]^T \tag{9}$$

as the filter state vector, and an exponential correlation model for $\ddot{r}$ as in [3] and [4], the optimal filter is altered very little by the presence of the coupling parameter Δt. In the terminology of Kalman filtering [5], the only alteration to the filter is in the matrix H of partial derivatives of the measurement with respect to the state variables,

$$H = [1 \;\; \Delta t \;\; 0], \tag{10}$$

whereas in the absence of coupling, H is simply $[1\ 0\ 0]$. (This alteration of H remains the only change, even in more complex Kalman filters, such as the reentry vehicle trackers of [6], for which results will be presented below.)

The basic operations which take place in the simple three-state "*ghk*" filter are those of prediction,

$$\hat{\mathbf{x}}_{i-} = \Phi(T_s)\hat{\mathbf{x}}_{i-1+}, \tag{11}$$

and correction,

$$\hat{\mathbf{x}}_{i+} = \hat{\mathbf{x}}_{i-} + K(m - H\hat{\mathbf{x}}_{i-}), \tag{12}$$

where the caret represents an estimate, the transition matrix $\Phi(T_s)$ is given by

$$\Phi(T_s) = \begin{bmatrix} 1 & T_s & T_s^2/2 \\ 0 & 1 & T_s \\ 0 & 0 & e^{-T_s/\tau} \end{bmatrix} \tag{13}$$

(or the more exact version given in [3]), and the dimensionless gain parameters g, h, and k, which are used to form the gain matrix

$$K = \begin{bmatrix} g \\ h/T_s \\ 2k/T_s^2 \end{bmatrix}, \tag{14}$$

may be computed as described in [5]. T_s is the sampling time (time between measurements), and the measurement m in (12) is now considered to be contaminated by an additive zero-mean Gaussian measurement error e with standard deviation σ_m:

$$m = \quad r + \dot{r}\Delta t + e \tag{15}$$

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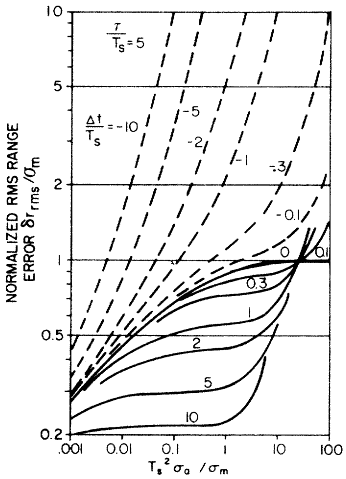

Fig. 1. Range estimation accuracy with range-Doppler coupling.

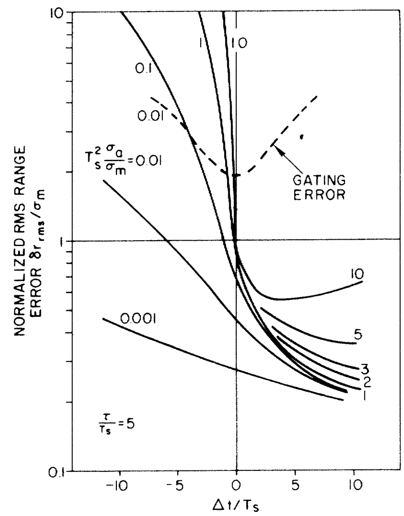

Fig. 2. Range accuracy and gating error versus $\Delta t/T_s$.

$$E(e) = 0 \tag{16}$$

$$E(e^2) = \sigma_m^2. \tag{17}$$

The range acceleration $\ddot{r}$ is assumed to be exponentially correlated, with rms value σ_a and correlation time τ,

$$\phi_a(t_1, t_2) = E[\ddot{r}(t_1)\ddot{r}(t_2)] = \sigma_a^2 \exp\left[-|(t_1 - t_2)|/\tau\right], \tag{18}$$

which determines the form of the transition matrix in (13).

It has been shown [4] that, in the absence of range-Doppler coupling, this simple problem is completely describable by the two dimensionless parameters

$$p_1 = \tau/T_s \tag{19}$$

$$p_2 = T_s^2 \sigma_a/\sigma_m. \tag{20}$$

When Δt is nonzero, it is necessary to introduce an additional parameter, which may be considered to be

$$p_3 = \Delta t/T_s. \tag{21}$$

It is apparent from the above that the presence of coupling slightly alters the computation of the input to the filter, the measurement "residual" in (12):

$$(m - H\hat{\mathbf{x}}_{i-}) = m - \hat{r}_{i-} - \hat{\dot{r}}_{i-}\Delta t. \tag{22}$$

The only other change is in the values of the gains g, h, and k. In general, positive Δt reduces g and h and increases k, while negative Δt has the opposite effect. For some values of the parameters p_1 and p_2, negative Δt may even cause g to exceed unity and/or k to become negative, neither of which ever happens when the coupling is absent.

Although the nature of the chirp measurement as a linear combination of range and range rate [as expressed in (4)] has been recognized for a long time [7], and although the corresponding alterations to the optimum tracking filter [see (10)] are quite trivial, the extent to which the range-Doppler coupling affects tracking accuracy does not seem to be generally appreciated. Hence, the primary purpose of this correspondence is to demonstrate these effects by means of numerical data. For the simple filtering problem described above, Fig. 1 presents curves of steady-state rms error in the position (range) estimate immediately after incorporation of a measurement (i.e., before prediction to the next measurement time). The parameter p_1 was kept constant at a typical value of 5, i.e.,

$$\tau = 5T_s, \tag{23}$$

and p_2 and p_3 were varied over wide ranges. The advantage of positive Δt is readily apparent. (Experience has shown that p_2 is usually less than about 5 to 10 in practice.) These curves are most useful for comparing positive and negative Δt's of the same magnitude, since a different magnitude of Δt generally means a different value of σ_m, unless some of the other waveform parameters are altered as well. In particular, if pulse length and power are kept fixed, σ_m is approximately inversely proportional to the swept bandwidth B, and, hence, proportional to Δt (assuming f_0 is unchanged). The curve for $\Delta t = 0$ can be interpreted as showing the performance which would be predicted by a simulation which erroneously ignores the existence of the coupling.

Fig. 2 reproduces the results of Fig. 1 plotted against $\Delta t/T_s$. Since prediction of $r + \dot{r}\Delta t$, rather than r alone, is the critical operation for range-gate setting, the error in this prediction provides a measure of tracking tenacity. Accordingly, Fig. 2 also presents one curve of normalized steady-state rms "gating" error,

$$(1/\sigma_m)[\delta(r + \dot{r}\Delta t)_{i-}]_{\rm rms} = (1/\sigma_m)[\delta r_{i-} + \delta\dot{r}_{i-}\Delta t]_{\rm rms}. \tag{24}$$

The conclusion to be drawn from this curve is that the gating accuracy, or tracking tenacity, does *not* depend on the

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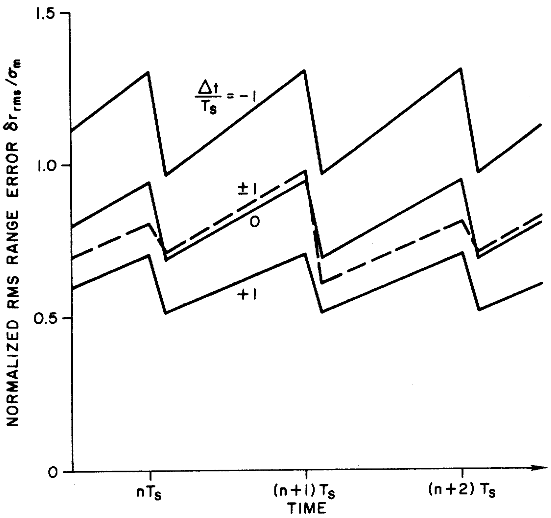

Fig. 3. Effect of alternating up- and down-chirp.

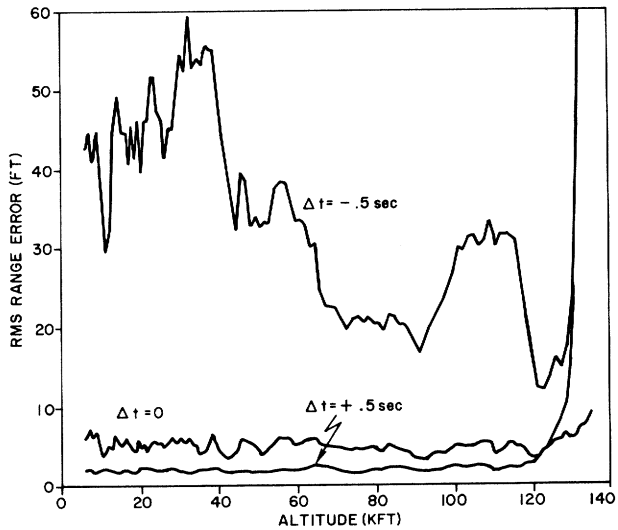

Fig. 4. Reentry vehicle range estimation accuracy.

sign of Δt. It is degraded, however, by the presence of any nonzero Δt, if σ_m can be assumed fixed.

The improved accuracy when Δt is positive can be explained, in a qualitative manner, by using the concept of "extrapolated range" [2]. When $\ddot{r}$ and/or Δt is small, the quantity $r + \dot{r}\Delta t$ is approximately equal to the range of the target at a time which differs by Δt from the actual measurement time. Thus, when Δt is positive, it is somewhat as though we were measuring the range at a future time. This yields better estimates at the present time (because interpolation is more accurate than extrapolation), and improved prediction because of the more timely data.

An alternative explanation can be offered in terms of cross correlations. Since a velocity error tends to propagate into a position error with the same sign, there tends to be a positive cross correlation between the position and velocity errors. This means that quantities of the form $r + c\dot{r}$, where c is a positive constant, are less accurately known than quantities of the form $r - c\dot{r}$. For the measurement to yield the most improvement, therefore, we should measure $r + \dot{r}\Delta t)$ with a positive Δt.

It has sometimes been suggested that improved performance might be achieved by the use of alternating up-chirp and down-chirp on successive pulses. This allows estimates of range and range rate to be made from two pulses only, and might be of some advantage in a system using a crude tracking algorithm. With the optimum tracking filter under consideration here, however, such an approach is apparently of little value. This is demonstrated by the comparative data of Fig. 3, which shows the steady-state "sawtooth" behavior of the normalized rms position error for the typical case $p_1 = 5, p_2 = 0.1$. (The jump of the sawtooth at each measurement has been spread over a finite time interval for clarity.) The alternating chrip case (broken line) yields rms errors which are alternately above and below those of the $\Delta t = 0$ case, but are always greater than those of the positive chirp case.

III. Reentry Vehicle Tracking

In order to examine the significance of this effect in a more realistic (and nonlinear) situation, range-Doppler coupling was added to a complete reentry vehicle tracking simulation. The tracking filter was very similar to the fully coupled seven-state range-direction-cosine filter described by Mehra [6]. The states estimated are three positions, r, u, v, three velocities, $\dot{r}$, $\dot{u}$, $\dot{v}$, and a function of the ballistic coefficient β (in this case ρ/β was used). The reentry vehicle initial conditions were

altitude = 135 000 *feet*

velocity = 24 000 *ft/s*·

flight-path angle = 35 *degrees*

and the impact point was located at the phased-array radar site. The ballistic coefficient β was 1500 *pounds per square foot*, plus a bias random from flight to flight, plus an exponentially correlated random process; this complex β model was assumed to represent random density variations, as well as actual ballistic-coefficient variations.

Measurements were taken ten times per second, and measurement accuracies (one-sigma) were approximately 1 *millisine* in angle and 8 feet in range (or, more correctly, in $r + \dot{r}\Delta t$).

Figs. 4 and 5 show rms range and range rate errors (from 25 Monte Carlo runs) for chirp time constants Δt of −0.5, 0, and +0.5 seconds. An order of magnitude difference can be observed between the positive- and negative-chirp cases. (Comparison with the zero-chirp case is less meaningful, since a change in $|\Delta t|$ with constant measurement accuracy implies changes in other waveform parameters.)

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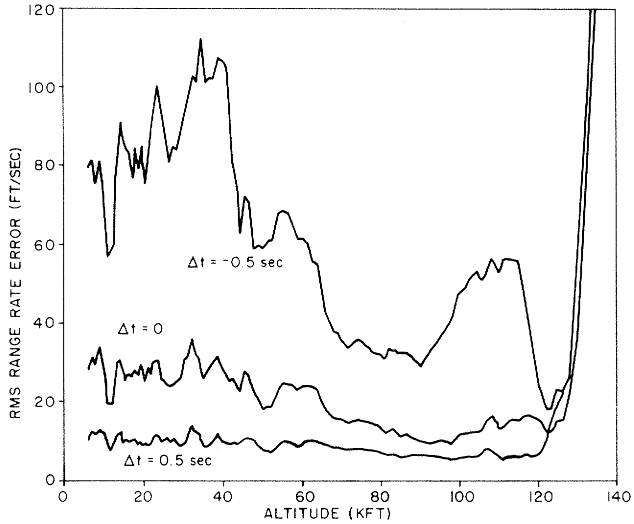
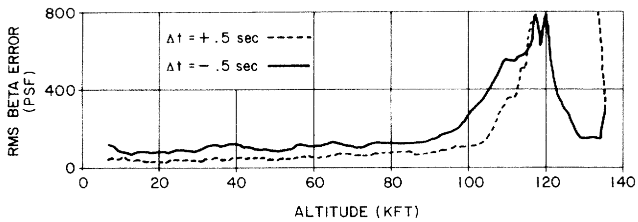


Fig. 5. Reentry vehicle range rate estimation accuracy.

Fig. 6. Ballistic coefficient estimation accuracy.



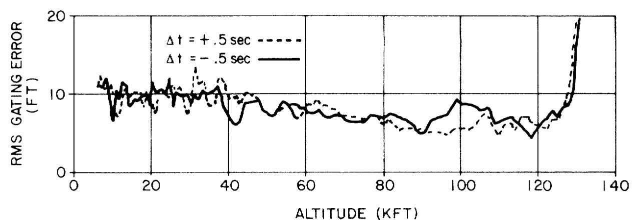


Fig. 7. Gating error for the reentry vehicle tracking problem.

Fig. 8. Intercept-point prediction accuracy from 90 000 feet.

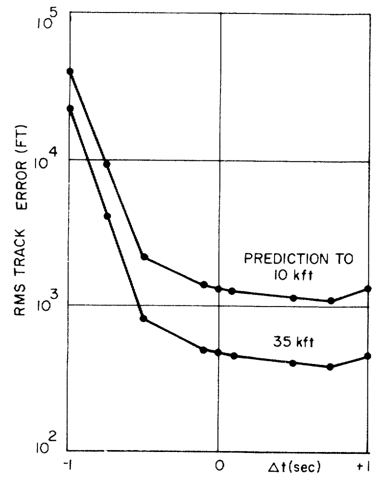


Fig. 6 shows the effect on ballistic coefficient estimation, and Fig. 7, which presents rms gating error [see (24)], verifies the conclusion of Fig. 2 that these are unaffected by the sign of Δt.

In the case of unfriendly reentry vehicles, an important aspect of the tracking problem is that of intercept-point prediction. Fig. 8 shows data for prediction from 90 000 feet to intercept altitudes of 35 000 and 10 000 feet. The error quantity plotted (versus Δt) is the rms value of the component of position prediction error along the velocity vector (which is the dominant component of the error). The figure seems to indicate that an *optimum* value of Δt exists; "*ghk*" filter studies have shown that this is indeed the case (Fig. 2), although the optimum often lies outside the range of reasonable values of the parameters.

IV. Conclusions

The main purpose of this correspondence has been to call attention to the existence of the range-Doppler coupling effect in chirp radars, and the surprisingly strong effect it may have on tracking accuracy. Its presence should not be overlooked in the design of tracking filters and in the prediction of their performance.

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Standardization of the Definition of the Radar Ambiguity Function

Abstract

A survey of the literature indicates an inconsistency in the definition of the radar ambiguity function. This correspondence suggests a definition consistent with Woodward's intent.

In his monograph[1] Woodward defined a correlation function $x(\tau, \vartheta)$ as

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