Critical Survey of Approaches to EWR Target Tracking: an endorsement of Kalman filters

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***Abstract*—New methodologies for target tracking and for evaluating its efficacy have recently emerged, all potentially being a "magic bullet". The questionable accuracy benefits, missing rigor (in some cases), and definitely large CPU-time computer loading drawbacks of the new estimation approaches are discussed as compared to a conventional Extended Kalman Filter and the novel Batch Maximum Likelihood Least Squares algorithm, as the well-known previous candidates for use in land-based Early Warning Radar (EWR) target tracking. A reminder is that the existing 30 year old Cramer-Rao lower bound evaluation methodology is already rigorous and adequate for evaluating target tracking efficacy in $P_d < 1$ situations when confined to exo-atmospheric target tracking, as arises in EWR. We also discuss the more challenging and sensitive angle-only filter methodologies, needed to handle target tracking when enemy escort jamming denies radar range measurements and impedes target tracking unless two or more radars synchronously triangulate to thereby enable joint tracking of enemy targets. Finally, supporting technologies (some old, some new) are discussed for enhancing the performance of these EKF's with only a modest increase in computational burden. Motivation for these pursuits is the quest to gain more veracity in on-line filter covariance calculation to mitigate any tendency to be overly optimistic (which could otherwise adversely affect multi-target track associations which typically utilize such EKF covariances within its initial gating stage).**

***Index Terms*—EWR Target Tracking, Filter Models, Kalman Filter, Approximate Nonlinear Filter, EKF, IEKF, Batch Least Squares, Unscented Filter, IMM Filter, Particle Filter, Covariance Fidelity, Probability One, Multi-target Tracking.**

I. Introduction and Overview

STRATEGIC target tracking typically employs a system dynamics target model that is nonlinear (with inverse square gravity along with its second zonal harmonic appearing in its associated ODE description, similar to system and measurement models described in detail in [26], [107], [218], [283], [297] with parameter values as provided, but updated in [114]). Radar target model is also nonlinear in the algebraic sensor observation equation, where range-Doppler ambiguity is compensated for within the plane of the antenna face [11], [294], needing a transformation algorithm [133] (that should be updated [134] to reduce the maximum error that can be incurred). Among the first cogent modern treatments of Reentry Vehicle (RV) target modeling for radar target tracking were[1] [289]-[291], [59]-[61] (and, afterwards, quickly followed by several others [62], [292]-[294]). EKF's are typically used for tracking the target state in a ballistic trajectory; but over the last 25 years, some form of nonlinear batch least squares (BLS) algorithm has been used for this purpose [135] on fast parallel processing machines by dynamically allocating and de-allocating memory, as needed. However, EKF's are still relied upon for the measurement intensive routine data associations arising in first forming the initial hypotheses within multi-target tracking (MTT) before switching to BLS for later track enhancement of *mature targets* as part of the overall MTT process. Typical behavior of target-associated confidence regions (CR) during tracking are conceptually depicted in Fig. 1, as they go from initially being a pancake (at horizon break, upon first entering the radar fence consisting of pre-programmed patterns of radar up-down pulse-pair chirps within multiple pencil beams sweeping and scanning for initial detections) to later being a football as more confirming radar sensor return "hits" are accumulated for this designated target. Target ID's are assigned by the radar controller/manager that subsequently schedules radar resources to enable further following of these objects of interest that are suspected of being potential threats after having passed a comparison test weeding them out from known satellites and space debris listed in an up-to-date Space Object Catalog (of 9,000+ entries)

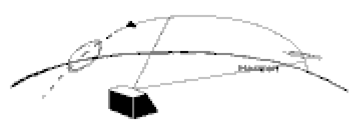

Fig. 1. Assumed Gaussian CR goes from being a pancake (at horizon break) to a football afterwards

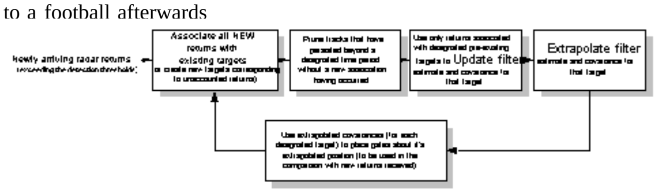

Fig. 2. Functional overview of the MTT Data Association Aspect of Track Maintenance [53]

Among the new approaches to target tracking and for evaluating its efficacy are unscented filters [1]-[4]; Covariance

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[1] Paralleling similar precedents by James R. Huddle (Litton), Stanley Schmidt and George Schmidt (Draper Laboratory), and Arthur Gelb (TASC) in recognizing the Kalman Filter's utility for Inertial Navigation Systems, circa 1967.

Intersection (CI) filters [5], [6]; Particle Filters (PF) [7], [8], [177]; and Farina et al's new formulation of Cramer-Rao Lower Bound evaluation for non-unity probability of detection (for $P_d < 1$) [9], all offering the lure of potentially being a "magic bullet". The questionable accuracy benefits, the missing rigor (in some cases), and definitely large computer loading drawbacks of the new estimation approaches all need to be considered (as discussed further in Secs. 2 to 5) before deciding to push forward to implementation for new EWR applications. We also review the older $\alpha - \beta$ filters as they have been recently reconsidered for EWR tracking [151].

Two of the new approaches were evaluated and cross-compared as being among the four reported in [10], but all with accuracy results obtained under a somewhat artificial scenario of use (viz., invoking only 4 tracking filter states and assuming only planar motion despite not being under the influence of central forces exclusively but projectile treated as if it were and the observing radar is only at the launch point and within the same plane as the target trajectory) thus leaving accuracy quantifications in [10] somewhat questionable as they relate to actual missile defense since this overly benign scenario lacks realism, as explained in Sec. 3. These approaches are discussed as compared to the standard load of two conventional Extended Kalman Filters like Range Velocity Cartesian Coordinates (RVCC), and R-U-V [11]-[13], and to novel Batch Maximum Likelihood Least Squares (BLS) algorithms [14], [15], [189, App. 2], as three well-known historical candidates for use in radar target tracking within land-based EWR. (Recall that Ref. [113] has already demonstrated that partitioned filters can be unsatisfactory in some situations and are therefore undesirable even for crossing targets.)

In Sec. 5, the existing 30 year old Cramer-Rao lower bound evaluation methodology [16]-[24] is shown to be rigorous and flexible enough to adequately evaluate target tracking efficacy in $P_d < 1$ situations for specified detection threshold settings when confined to exo-atmospheric interception of a target and its prior tracking, as arises in EWR, where truth model process noise is theoretically zero. The *filter model* can have nonzero process noise tuning [2] and still abide by this $Q = 0$ constraint dictated by the *truth model's structure*. However, Farina et al's formulation [9] (based on [25]) is useful for evaluating Indo-atmospheric tracking, which is a more stressing situation, where system truth model process noise is definitely nonzero (reflecting atmospheric buffeting associated with reentry drag, or maneuvering, or with a projectile undergoing late stage thrusting).

As discussed in Sec. 6, motivation for these pursuits is the quest to gain more veracity in on-line filter covariance calculation to mitigate their tendency to be overly optimistic (or, rarely, pessimistic) since any hand-over or multi-target (MTT) associations rely on their veracity (Figs. 2, 6) and errors in the values of these covariances (being the only ones actually available in one-shot real world trials) are sensitive. In Sec. 7, we recommend pursuit of good tracking accuracy with probability one success (as pioneered by the late Frank Kozin in the 1960's and 1970's) over current Monte-Carlo-based mean square averaging techniques, only appropriate for aggregate behavior.

TABLE I
RELATIVE CPU BURDEN OF 4 DIFFERENT ALGORITHMS [59, P. 44]

EKF	$\alpha - \beta - \gamma$ (or a-b-c)	EKF (adaptive)	2^{nd} order Filter	Iterated (IEKF)
1	0.1	1.3	1.5	2.0

In Sec. 8, we discuss the more challenging and more initial-condition-sensitive angle-only tracking (AOT) filter methodologies [28]-[33], needed to handle target tracking when enemy escort jamming denies the radar its target range measurements and thus impedes tracking.

After reviewing the limitations of earlier trackers, supporting technologies (some old, some new) are discussed in Sec. 9 for enhancing the performance of EKF's, which incur only a modest increase in the computational burden (all applicable as evolutionary low risk enhancements to the EKF's already present in EWR) as variations that more easily satisfy hard real-time constraints.

We offer our views here based on past experience in the theory [3] and applications of estimation, tracking, and Kalman filtering [43]-[58], [68], [75], [76] and by using the strong tradition of quantifying the relative computer burdens of various filter alternatives beforehand [59]-[63] for sequential implementations [4] using an awareness of the characteristics of what constitute acceptable exact and approximate solutions to nonlinear estimation problems [64] (cf., [65]), [73], [282], [285], [286]). We reference older surveys here [259], [258] [5] as respected precedents because of their correctness (see Table 1). A newer published survey [69] by Lincoln Lab in 1984 (specifically for RV target tracking) found no significant changes from the same approaches and 1970 principles of [59] other than looking at evolutionary changes and that the 1970 ranking of candidate tracking filter approaches hadn't changed much (or expanded, despite hundreds of subsequent researchers tackling the problem). Evidently, others also fear that many of the very recent new alternative approaches to the use of EKF's for target tracking are over-hyped (see [86, Sec. VI], [102], [112]). Further evidence supporting this view is offered here in Secs. 3, 4, and 9 but we are receptive enough to also report all benefits. Constructive evolutionary EKF developments that have occurred in the last decade to

[2] A quantitative rationale is offered in [284] for the appropriate magnitude of compensating ficticious process noise utilized in *tuning* an EKF filter model that seeks to track exo-atmospheric ballistic targets.

[3] The theory of nonlinear filtering, in general, requires familiarity with Ito, Stratonovich, and McShane integrals as well as an understanding of measure-theoretic probability (including nested expanding sub-sigma algebras and conditional expectations with respect to them, martingales and associated inequalities, and *law of the iterated logarithm*).

[4] The CPU burden for sequential implementation is used merely as a baseline cross-check for eventual parallel implementation in modern day hardware consisting of either embedded processors or powerful parallel processing mainframes, where it is expected that considerable speed-up should accrue (but does not always initially occur because of processing bottlenecks that need to be identified and removed until the expected speed-up is achieved). Historically, many so-called fast parallel versions of famous algorithms were unexpectedly slower than their sequential counterparts.

[5] While Prof. Thomas Kailath (Stanford) is almost always in the right, a notable exception was [262] vs. [263].

improve performance are pointed out (especially in Sec. 9) and we also point out other improvements, some as mundane as merely identifying new best parameter values [107], best practices for radar target tracking [114] (and sometimes as best models for INS/GPS navigation [271] that also plays a useful role in locating moving antennas or in serving as a source of *true* position and velocity for targets equipped with GPS translators), and in making the derivation of results simpler and more straight forward [181], [270]. Novelty and creativity are always encouraged but we also strongly desire that the test conditions for algorithm evaluations be realistic and actually representative of the application scenario.

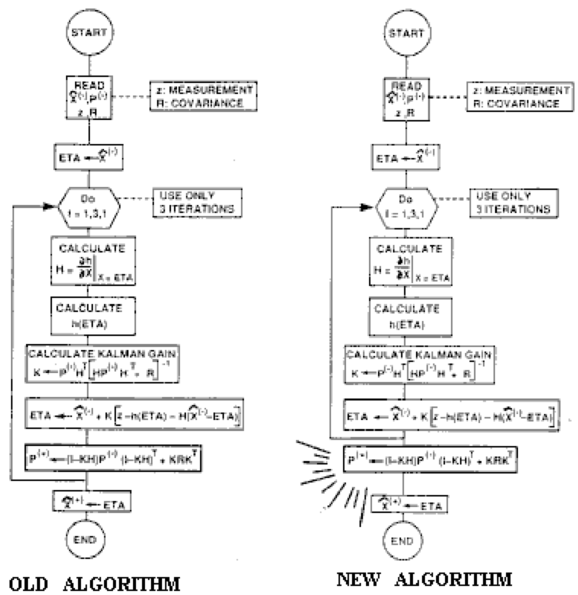

Fig. 3. Our new approach reduces the computational burden of Iterated Extended Kalman Filtering (from [35])

In 1989, we were able to make a slight change in the ranking of an Iterated EFK [35] [6] by simplifying its computer burden (Fig. 3). Instead of an IEKF being twice the computed load of a comparable EKF, as [59] reported (see Table 1), we offered an evolutionary modification and improvement that made our IEKF just $1.33\bar{3}$ the computer load of a comparable EKF yet yielded identical results of the same accuracy as the earlier more computationally intensive version. Also see [122]. (Carlson's 1973 version of Squareroot filtering now beats Gerald Bierman's later 1975 $U-D-U^T$ formulation merely by the way computer processor hardware and its associated firmware algorithms are now implemented [58, App.]; prior to the mid 1990's, it was vice-versa. However, squareroot filters may not even be needed for tracking RV's since a specific designated target is only in view for less than 30 minutes and the discrete measurements received may be somewhat sparse and relatively fewer than is usually the case for comparable Navigation applications, where frequent periodic measurements warrant use of numerically stable squareroot filters to compensate for the round-off error associated with the relatively more frequent opportunities for measurement incorporation into tracking filter updates.)

Profs. R. E. Mortensen, H. W. Sorenson, E. B. Stear, A. B. Stubberud, and R. C. Kolb hosted the (U.S. Air Force sponsored) Nonlinear Estimation and Its Applications Conference from 1970 until it ended in 1975. Although many innovative, well-funded researchers were working hard on the nonlinear filtering problem, this conference was canceled because significant new results usually don't accrue yearly [7] for the hard problem of nonlinear filtering. Even today, some of the best insights and early leads for tackling nonlinear filtering are found in these past proceedings (e.g., [129], [130]). Recommendation for future work in Secs. 9 and 10 logically continue where our predecessors left off by emphasizing existing barriers that still need to be tackled and conquered rather than suggesting a search for new approaches just because they are new (while apparently ignoring past problems already elucidated as needing solving). A brief summary and further perspective on goals accomplished here are provided in Sec. 11. For those readers desiring a refresher or more of an introduction, a brief summary of Kalman Filtering appears in Appendix A, which emphasizes software architecture needed to appropriately match the structure of a particular application.

II. Drawbacks of Covariance Intersection

A counterexample is presented to a result claimed in a proof in [6], pertaining to using this new approach to Covariance Intersection (CI). Other researchers have already demonstrated certain problems that exist with earlier versions of CI, as summarized from a survey [74] of the previous CI approaches encountered in Target tracking applications. We alert readers to investigations along similar lines from the field of navigation that were apparently overlooked in [74] that convey similar but different results for ascertaining ellipsoidal overlap and for combining the estimates from two or more Kalman filters,

[6]The main focus in [35] is the structure and performance of an IEKF vs. an EKF for RV tracking, even though the oversimplified model common to both is acknowledged to not include J_2 (which, when present, accounts for the earth's oblateness) nor is Doppler compensation in the face of the antenna present, as is now known to be desirable to include both of these terms in a tracker's model since they are needed for more realism in modeling Missile Defense situations [14], [15], [53] because greater tracking accuracy is reaped as a consequence of their presence in the model.

[7]With the notable exception of F. E. Daum's new nonlinear filtering results in '86 (IEEE AC), '86 (ACC), '86 (20^{th} *Conf. on Inform. Sciences and Systems*, Princeton), '87 (IEEE AC), '88 (Ch. 8, ed. by J. C. Spall, Marcel Dekker) '94 (SPIE, Orlando, FL), '97 (SPIE, San Diego, CA), '01 (*Proc. of Tribute to Y. Bar-Shalom*), '03 [8] but none yet applied except for an application with only 4 planar states [310] that, perhaps, may be considered too overly simplistic to be practical and Daum's later important results are still infinite dimensional (viz., in general, requiring full integration of a non-Gaussian pdf at each update step) so they are not computable in real-time (unless computed beforehand and stored off-line, a procedure found undesirable in the 1970's when measurements did not arrive exactly when expected, as had been previously planned). An exception is that all of these results simplify to a tractable real-time Kalman filter when both process and measurement noises are Gaussian and the system is merely linear, as do several other existing historical 40+ year-old nonlinear filtering approaches as a precedent, notably (1) solving the Fokker-Planck Partial Differential Equation (also known as the forward Kolmogorov equation) for the evolution in time of the conditional pdf of the system state directly or, equivalently, (2) taking its Fourier Transform to obtain a conditional Characteristic Function, from which the random processes' conditional moments can be generated via differentiating it a requisite number of times (where both these approaches benefit from the additional structurural simplification of encountering certain noises from an *exponential family* possessing Gaussian conditional pdf's [180]), where these minor generalizations are now detailed in TK-MIP ©.

each representing a different sensor's output that has been previously processed. In all cases, CI is practically useless despite having beautiful analytic proofs.

The CI-based sensor fusion methodology of [6] usually degenerates to cases where $\mu = 0$ or $\mu = 1$, rather than the more useful situations where $0 < \mu < 1$. Some of the blame should be shared by IEEE reviewers of [6], who allowed it to be published even though there were no numerical parameters specified for the single diagram that appeared within, which was merely conceptual, and the single numerical example only illustrated the degenerate case of $\mu = 0$ or $\mu = 1$ (not the useful case where $0 < \mu < 1$, which is seldom met).

A. *Introduction/overview of the newer approach to CI*

This technical note offers a counterexample to the use of the results of [6] in this new Covariance Intersection (CI) approach. An expression for the estimate that results from combining two prior (assumed) independent estimates consisting of $(\hat{x}_1, P_{aa})$ and $(\hat{x}_2, P_{bb})$ is of the following well-known form ([165], and as summarized from Eq. 2 to the end of Sec. II of [6]):

$$\hat{x}_c = K_1\hat{x}_1 + K_2\hat{x}_2 = P_{bb}P_{cc}^{-1}\hat{x}_1 + P_{aa}P_{cc}^{-1}\hat{x}_2 = P_{aa}^{-1}P_{cc}\hat{x}_1 + P_{bb}^{-1}P_{cc}\hat{x}_2. \quad (1)$$

The corresponding exact covariance for the above, with the *assumption of possessing unbiased estimates* throughout, is:

$$\tilde{P}_{cc} \triangleq E[\tilde{x}_c\tilde{x}_c^T], \text{ where } \tilde{x}_c \triangleq x_{\text{true}} - \hat{x}_c. \quad (2)$$

The above expression of Eq. 1, consisting of the indicated weighted combination of the two prior linear estimates and utilizing the accompanying covariances P_{aa} and P_{bb}, seeks to use an acceptable approximate covariance P_{cc} that conservatively suffices in its role of making Eq. 1 be a useful single combined estimator if and only if P_{cc} is a consistent covariance (in the matrix positive semi-definite sense) by satisfying the following required upper bound criterion ([6, Eq. 4]):

$$P_{cc} \geq \tilde{P}_{cc}, \quad (3)$$

and the quest for a satisfactory *consistent covariance* upper bound motivated use of this particular expression:

$$P_{cc}(\omega) = [\omega P_{aa}^{-1} + (1-\omega)P_{bb}^{-1}]^{-1}, \quad (4)$$

(advocated for use in Eqs. 9 and 10 of [6]), which when substituted back into Eq. 1 yields:

$$\begin{aligned}\hat{x}_c &= K_1(\omega)\hat{x}_1 + K_2(\omega)\hat{x}_2 \\ &= \omega[\omega P_{aa}^{-1} + (1-\omega)P_{bb}^{-1}]^{-1}P_{aa}\hat{x}_1 \\ &\quad +(1-\omega)[\omega P_{aa}^{-1} + (1-\omega)P_{bb}^{-1}]^{-1}P_{bb}\hat{x}_2. \quad (5)\end{aligned}$$

Ref. [6] then advocates optimizing ω in the above to minimize the trace of Eq. 4 (cf., [6, Eq. 14]):

$$\text{tr}\left(P_{cc}(\omega)\right) = \text{tr}\left([\omega P_{aa}^{-1} + (1-\omega)P_{bb}^{-1}]^{-1}\right), \quad (6)$$

and goes further to provide Theorem 2 [6, p. 1881] that claims the global minimum occurs for $\omega^* \in [0,1]$. The resulting optimized ω^* is then substituted back, respectively, into the expressions of Eqs. 4 and 5 (even when the constituent component estimates are no longer independent and the cross-covariance P_{ab} may be unknown or inaccessible) to be the best fused estimate of the form:

$$\begin{aligned}\hat{x}_c &= K_1^*\hat{x}_1 + K_2^*\hat{x}_2 = \omega^*[\omega^* P_{aa}^{-1} + (1-\omega^*)P_{bb}^{-1}]^{-1}P_{aa}\hat{x}_1 \\ &\quad +(1-\omega^*)[\omega^* P_{aa}^{-1} + (1-\omega^*)P_{bb}^{-1}]^{-1}P_{bb}\hat{x}_2, \quad (7)\end{aligned}$$

with the corresponding accompanying associated covariance:

$$P_{cc}^* \triangleq P_{cc}(\omega^*) = [\omega^* P_{aa}^{-1} + (1-\omega^*)P_{bb}^{-1}]^{-1}. \quad (8)$$

Since the original two estimates and accompanying covariances are all real quantities, clearly, the two expressions of Eqs. 7 and 8 also need to yield exclusively real results. If one were to obtain a complex answer for ω^* as the solution that globally minimizes the criterion of Eq. 6, this would constitute a counterexample to what is claimed and ostensibly proved in Theorem 2 [6, p. 1881], namely, that the optimizing ω^* either lies on the two boundary points 0 or 1 or lies within the interior of $[0,1]$. Once this was *definitively established* according to [6], they could then turn their attention in [6] by just searching over the compact interval $[0,1]$ for the minimum that is guaranteed to exist from first principles of real analysis for this continuous cost criterion of Eq. 6 (as the composite of the trace and the matrix inverse) since scalar continuous functions always achieve both a maximum and a minimum on a compact set. However, only Theorem 2 of [6] asserts that such a minimum is also the global minimum (otherwise it would not be of interest since this criterion of using the trace of the associated covariance was specifically chosen within [6] to be consistent by dovetailing with what was already correspondingly used in the derivation of the underlying Kalman filters from which the prior constituents (x_1, P_{aa}) and (x_2, P_{bb}) were obtained). If this local minimum were indeed also the global minimum, we would have no further objections here. However, Example 1 below serves as a counterexample to the global optimization assertion [6, Thm. 2] since it yields a complex answer for ω^*.

Similarities and connections to other tests for ellipsoid overlap and pre-existing warnings regarding other earlier Covariance Intersection approaches are discussed in Sec. 2.4.

B. *A numerical counterexample*

A closed-form evaluation will now be provided for this new version of CI [6] for the simple numerical example below that exposes a difficulty with using this CI approach that has not been previously publicized.

Example 1:

$$P_{aa} = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}; P_{bb} = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix};$$
$$P_{bb} - P_{aa} = \begin{bmatrix} 1 & 1 \\ 1 & 7 \end{bmatrix} > 0, \text{ and } [P_{bb} - P_{aa}] \text{ has } \lambda = 2,\ 6. \quad (9)$$

Notice that both P_{aa} and P_{bb} above are positive definite, as with all non-degenerate covariances.

To explicitly demonstrate this new CI technique of [6], we seek to apply the solution of Problem 2 (of [6]) to Eq. 9 above yielding the following abbreviated intermediate steps as we seek to directly solve for the optimizing ω^* that should minimize the trace of P_{cc} below (according to the procedure of [6, Eq. 16] using the derivative convention stated in the footnote below):

$$P_{aa}^{-1} = \begin{bmatrix} \frac{8}{15} & \frac{-2}{15} \\ \frac{-2}{15} & \frac{8}{15} \end{bmatrix}; P_{bb}^{-1} = \begin{bmatrix} \frac{12}{33} & \frac{-2}{33} \\ \frac{-2}{33} & \frac{4}{33} \end{bmatrix}; \quad (10)$$

$$P_{cc} = \begin{bmatrix} \frac{8\omega}{15} + \frac{12(1-\omega)}{33} & \frac{-2\omega}{15} - \frac{2(1-\omega)}{33} \\ \frac{-2\omega}{15} - \frac{2(1-\omega)}{33} & \frac{8\omega}{15} + \frac{4(1-\omega)}{33} \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \frac{84\omega+180}{495} & \frac{-36\omega-30}{495} \\ \frac{-36\omega-30}{495} & \frac{204\omega+60}{495} \end{bmatrix}^{-1} = \frac{165\begin{bmatrix} (68\omega+20) & (12\omega+10) \\ (12\omega+10) & (28\omega+60) \end{bmatrix}}{\text{DENOM}}; \quad (11)$$

$$\text{tr}[P_{cc}] = \frac{165[(68\omega+20)+(28\omega+60)]}{\text{DENOM}}, \quad (12)$$

where DENOM$\stackrel{\triangle}{=} (28\omega+60)(68\omega+20)-(12\omega+10)^2$. The critical points of the above trace are obtained from setting $\frac{\partial}{\partial\omega}\text{tr}[P_{cc}] = 0$ and, after simplifying, solving for the zeros of:

$$0 = 42,240\omega^2 + 70,400\omega + 61,600, \quad (13)$$

a quadratic equation; with solutions being:

$$\omega^* = \frac{-70,400 \stackrel{+}{-} \sqrt{(70,400)^2 - 4(42,240)(61,600)}}{2(42,240)} = \frac{-70,400 \stackrel{+}{-} \sqrt{-5,451,776,000}}{2(42,240)}. \quad (14)$$

The above Eq. 14 possesses no solutions [8] over the real field and, in particular, has no solution within the predicted interval $[0, 1]$ and so the new CI approach of [6] is apparently stymied here and can proceed no further for this numerical example corresponding to $[P_{bb} - P_{aa}]$ being strictly positive definite. We did not anticipate that this new CI approach of [6] would have such problems when the containment condition demonstrated in Eq. 9 (i.e., $P_{bb} - P_{aa} > 0$ or, equivalently, $P_{bb} > P_{aa}$) was strictly met (a condition that was present but down played in the proof of [6]); so we were surprised when it failed to yield an adequately real solution for ω^*.

While it is indeed true that a continuous function of ω (such as the matrix inverse, constituting the RHS of Eq. 4, composed with the trace operation of Eq. 6) over a compact interval like $[0, 1]$ achieves its minimum there, we reject the suggestion that we merely confine optimization to be over $[0, 1]$ since such a constraint would, in general, only yield a local minimum. The proofs of [6] supposedly guarantee that by merely optimizing the expression of the RHS of Eq. 6 over just the interval $[0, 1]$, the result would also be the global minimum. However, this non-pathological Example 1 above demonstrates this claim of [6] to be false.

According to [6], only after minimizing the above Eq. 12 can the two optimal gains and resulting associated optimal covariance P_{cc} be explicitly evaluated (by substituting the result of Eq. 14 back into Eqs. 7 and 8, respectively, where Eq. 8 has already been simplified to be Eq. 11) using CI. The above numerical example exhibits a result that is therefore inconsistent with what the CI approach of [6] asserts (contrary to what is expected as supposedly proved in [6, Thm. 2]) so [6] appears to not work as it should in all cases.

By insights availed from [67, p. 1141], it is recognized that for two ellipsoids sharing a common center, the covariance inclusions (such as that depicted in Eq. 18) serve as a test for full containment of one ellipsoid within another if and only if the matrix difference between two covariance matrices is positive definite. Numerical tests for positive definiteness/semi-definiteness are well known [48] and can serve as a warning of this same condition where the approach of [6] will likely fail, as depicted here in the numerical example above.

While it can be argued that, initially, there is no apparent physical reason why these initial covariance matrices should exhibit any partial ordering between them. Two synchronized decentralized estimates of the same target state vector, as viewed from different sensors with, perhaps, different perspective views, different segments of the electromagnetic spectrum utilized (to exploit inherent target characteristics), and different noise contamination intensities is but one example of why unaltered initial covariances would not necessarily exhibit such a partial-ordering in a completely general sensor fusion application but, instead, likely be skewed off from each other in tilt and overall size. However, use of conventional radar along with a collocated laser radar may yield one target ellipsoid contained entirely within another due to the greater resolution (due to shorter wavelength) and generally smaller azimuth error incurred for laser optics.

C. Offering a simpler CI interpretation based on a different matrix inequality

A simpler approach is now explored here, based on convexity of the matrix inverse over positive definite matrices [66], as:

$$[\omega A + (1-\omega)B]^{-1} \leq \omega A^{-1} + (1-\omega)B^{-1} \text{ for all } 0 \leq \omega \leq 1. \quad (15)$$

When this result is applied to the expression of Eq. 4 above in seeking a covariance upper bound as in Eq. 3, the following results, requiring no matrix inversions at all for the RHS vs. a LHS (from [6, Eq. 4]) that does:

$$P_{cc} \stackrel{\triangle}{=} [\omega P_{aa}^{-1} + (1-\omega)P_{bb}^{-1}]^{-1} \leq \omega P_{aa} + (1-\omega)P_{bb} \stackrel{\triangle}{=} P'_{cc} \text{for all } 0 \leq \omega \leq 1. \quad (16)$$

Notice that P'_{cc} on the RHS represents an upper bound that is easier and more convenient to obtain and, moreover, by performing trace operations throughout Eq. 16, also yields a corresponding simple upper bound on the trace of P_{cc} as:

$$\text{tr}[P_{cc}] = \text{tr}[\omega P_{aa}^{-1} + (1-\omega)P_{bb}^{-1}]^{-1} \leq \omega\text{tr}[P_{aa}] + (1-\omega)\text{tr}[P_{bb}] = \text{tr}[P'_{cc}] \text{for all } 0 \leq \omega \leq 1. \quad (17)$$

However, although it is rigorous, this path is not a panacea since the resulting bound is likely to be slightly coarser (i.e., larger), in general than what would be provided by

[8] We formed $\frac{d}{dw}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dw} - u\frac{dv}{dw}}{v^2}$ and set $v\frac{du}{dw} - u\frac{dv}{dw} = 0 \Leftrightarrow u\frac{dv}{dw} - v\frac{du}{dw} = 0$, and so Eq. 13 here was effectively multiplied throughout by -1, but that still preserves the location of the roots of the resulting quadratic equation.

the optimizing CI approach of [6] (in situations where the approach of [6] in fact works, which is exceedingly rare). The benefit of this alternate approach is that (1) it requires no matrix inversions at all in its numerical evaluation and (2) it is always true for all without any qualifications. The convexity property itself delineates the interval of primary interest to be $[0, 1]$ and not because of some tenuous auxiliary theorem, as with [6]. The procedure of [6] cannot be applied for this example since [6, Thm. 2] is evidently violated.

D. Conclusions, summary perspectives, and beyond

Uncertainty being summarized as covariance ellipsoids normally only rigorously arises for the case of standard linear systems with Gaussian initial conditions independent of the additive Gaussian process and measurement noises (with known covariance intensities) and outfitted with a pure Kalman filter as an optimal linear estimator, mechanized either in a decentralized or centralized manner. Ellipsoidal confidence regions of constant pdf would also reasonably represent the class of elliptical distributions [9] and the conditional and marginal distributions of the exponential family of distributions but they typically do not arise (so far) in the standard estimation and filtering context of most normal target tracking or navigation applications.

Caution is conveyed here regarding the result of [6] apparently not applying when one ellipsoid is wholly contained within the other. An apparent hole in the applicability of the Covariance Intersection (CI) approach of [6] was illustrated here using an explicit numerical example. A coincidence is that the two participating covariances being related as

$$P_1 < P_2 \tag{18}$$

was historically encountered by this author in [43] before being able to specify a test for ellipsoid overlap (in n-dimensions) when the centers of the respective ellipsoids differ, where the particular covariance matrix, P_1, in [43], the solution of the Riccati equation is so related to the other covariance matrix, P_2, in [43], the solution of the Lyapunov equation. Remarkably, the result of [43] parallels (but is not identical to) what is done in [6]. However, the proof of Eq. 18 holding for the application of [43] was easily accomplished in Lemma 5.1 of [43] by just taking the synchronous difference of the two respective matrix differential equations that describe their evolution in time (in either continuous- or discrete-time) by demonstrating that the difference is always positive definite (as it evolves for all time steps $k > 0$) as the positive definite matrices, as pre- and post-multiplied by an indicated nonsingular matrix and its transpose (yielding a positive semi-definite intermediary matrix) and added to a positive definite matrix yielding a positive definite matrix result as in [43, Lemma 5.1] and in [244] (cf. [287, Lemma 2]).

The associated optimization problem in [6] has great similarity to that in [43] since the associated Lagrange multiplier was also merely a scalar. In the case of posing the simpler problem of a one dimensional test for the overlap of scalar Gaussian confidence intervals in [55] to show how the same test then generalizes to n-dimensions in [43], as a test for the overlap of Gaussian Ellipsoidal Confidence Regions, the version of the test in [55], [140] (being simpler than that in [43]) reveals other aspects that are similar in form to the structure encountered in [6] in enabling a closed-form answer to the optimization that also proceeds in both [43] and [55], after just optimizing the selection of λ^* along a scalar direction (i.e., the essence of the main result of [6]). When the containment condition is strictly satisfied, the numerical example of Sec. 2.2 failed to satisfy the expected condition on the optimum value of ω^* that it fall somewhere within the real interval $[0, 1]$. (Ref. [6] also lacks any corresponding numerical description or, alternatively, any explicit reference for the illustrative planar examples presented in Figs. 1 and 2 of [6], respectively, of the intersection inscribing and circumscribing ellipses that supposedly motivates how their approach should behave.)

A scalar example is now offered that should convince the reader of the problem with CI. For a scalar case situation, the ideal formula for the associated covariance of two fused estimates, when the two underlying constituent estimates are independent, looks like the familiar formula from electric circuit theory for combining two resistances in parallel (and is known to result in an answer that is less than the smaller of the two). This result is intuitively appealing and consistent with the tenets of Kalman Filtering. The following two algorithms: (1) the CI covariance of [6] using any ω, and (2) the alternative expression for the covariance, offered in Eq. 17 as a new result (using convexity of the matrix inverse over positive definite matrices) both yield a covariance that is larger than the smallest of the two original covariances unless ω is either 0 or 1, in which case it has a resulting covariance that is identical to the smaller one.

<u>Case I</u>: For two ideal independent estimates, the resulting covariance for the combined estimates would be:

$$\tilde{p}_{cc} = \frac{1}{\frac{1}{p_1}+\frac{1}{p_2}} = \frac{1}{\frac{(p_2+p_1)}{p_1 p_2}} = \frac{p_1 p_2}{(p_2+p_1)} = \left\{ \begin{array}{l} \frac{p_2}{\left(\frac{p_2}{p_1}\right)+1} \le p_2, \text{ for } p_1 > 0,\ p_2 > 0 \\ \frac{p_1}{\left(\frac{p_1}{p_2}\right)+1} \le p_1, \text{ for } p_1 > 0,\ p_2 > 0 \end{array} \right\} \le \min\{p_1, p_2\} \tag{19}$$

<u>Case II</u>: For the new CI algorithm of [6] in Eq. 4 for $0 < \omega < 1$, the covariance for the fused estimates would be:

$$\tilde{p}_{cc} = \frac{1}{\frac{\omega}{p_1}+\frac{(1-\omega)}{p_2}} = \frac{1}{\frac{(\omega p_2+(1-\omega)p_1)}{p_1 p_2}} = \left\{ \begin{array}{l} \frac{p_1 p_2}{(\omega p_2+(1-\omega)p_1)} \le \frac{p_1 p_2}{(\omega p_2+(1-\omega)p_2)} \le p_1, \text{ when } 0 < p_2 \le p_1 \\ \frac{p_1 p_2}{(\omega p_2+(1-\omega)p_1)} \le \frac{p_1 p_2}{(\omega p_1+(1-\omega)p_1)} \le p_2, \text{ when } 0 < p_1 \le p_2 \end{array} \right\} \le \max\{p_1, p_2\} \tag{20}$$

<u>Case III</u>: For the CI alternative path of Eq. 16, based on convexity of the matrix inverse over positive definite matrices, for $0 \le \omega \le 1$, the covariance for the fused estimates would be:

$$\tilde{p}_{cc} = \frac{1}{\frac{\omega}{p_1}+\frac{(1-\omega)}{p_2}} \le \omega p_1 + (1-\omega)p_2 = \left\{ \begin{array}{l} \omega p_1 + (1-\omega)p_2 \le \omega p_2 + (1-\omega)p_2 = p_2, \text{ when } 0 < p_1 \le p_2 \\ \omega p_1 + (1-\omega)p_2 \le \omega p_1 + (1-\omega)p_1 = p_1, \text{ when } 0 < p_1 \le p_1 \end{array} \right\} \le \max\{p_1, p_2\} \tag{21}$$

[9] *Elliptical distributions* have recently been used by Muralidhar Rangaswamy (IEEE Fellow, AFRL, Hanscom AFB) in attempting to compensate for the ground clutter seen by airborne radar.

So both these latter two CI covariance calculation paths yield the same answer for the scalar case! (Convexity of the inverse for positive definite matrices is rigorously established elsewhere, with many precedents in the engineering and mathematical literature cited in [66].)

From the above Cases II and III with similar manipulations, it is also easy to show that $\min\{p_1, p_2\} \leq p_{cc} \leq \max\{p_1, p_2\}$ and, likewise, that $\min\{p_1, p_2\} \leq p'_{cc} \leq \max\{p_1, p_2\}$. Please compare the last two results above to the tighter result of Case I above for 2 independent Kalman filter estimates. This is not a very satisfying outcome of applying either of these two CI strategies. Thus we illustrate the lack of appeal of either of these CI expressions to data fusion.

Ref. [74] is an excellent overview, in depth numerical ranking, and clear interpretation of all of the various approaches to sensor fusion that have occurred in the target tracking literature over the last two decades and culminates in an algorithmic improvement to CI that [74] attributes to the pioneering work of the late Fred C. Schweppe's unknown but bounded approach [77], where, instead of embracing the assumption of Gaussian noises being present throughout, uses an entirely deterministic approach of circumscribing the set of potential outcomes arising at each discrete-time step of a linear system's output within an ellipsoid. Schweepe's bounding ellipsoid approach, although creative, was notoriously conservative and was never used in applications nor was it admired as frequently as what is reported on in [42], as more representative of Schweppe's genius. (Overlooked in [74], parallel developments were occurring in navigation, as similar techniques were being examined, e.g., [68], [75], [76]; some criticized by Larry Levy JHU/APL [78]; but refuted in [52] as pertains to Jason Speyer's exact 1979 version of decentralized estimation [193] (also see [79]). Ref. [74] demonstrates that the approximate algorithms of the CI approach should only be used with extreme caution since uncertainty increases as more measurements are used (and fused) as a counterintuitive, extremely unsettling result stated and proved simply and convincingly in [74, following Eq. 22]. Apparently CI also has round-robin Web sites [5] in 2005, where one site refers to the other for missing details and vice versa, yet the specific details are never supplied.

Please see Ref. [80] for an elegant solution to the problem of assessing whether two 3-dimensional ellipsoids overlap that is both easy to understand and apparently straightforward to test for numerically. [10] Problems with [80] and a more straightforward approach to applying its results to determine overlap are in follow-up discussions in [244] and [245].

Before proceeding, a distinction is made here between what is offered in [80] and what is offered in [66] as a test for ellipsoid containment before making other historical connections and observations. Ref. 2 provides a test for full containment of one ellipsoid within another only when they share a common center, $\bar{x}$, as between $\frac{(x-\bar{x})^T P_1^{-1}(x-\bar{x})}{2} \leq 1$ and $\frac{(x-\bar{x})^T P_2^{-1}(x-\bar{x})}{2} \leq 1$ and the second is fully contained within the first if and only if the condition of Eq. 18 holds as a strict positive definiteness condition on matrices which themselves are each positive definite (as are all well behaved, non-degenerate covariance matrices [48]).

The overlap test of Ref. 80 needs matrix positive definiteness/semi-definiteness tests along with an implied eigenvalue-eigenvector calculation. Ref. [80]'s test is obtained by exploiting features of a (3-D)-ellipsoid translation represented as a rotation in 4-D space, a technique familiar in computer graphics applications [166, pp. 479-481].

By not needing any condition of Eq. 18 to be satisfied, Ref. [80] is for a more general case than treated in [43], [55], [140], [167]; however, the numerical calculations of [43], [55] are tailored for a standalone real-time decision (that was used aboard U.S. submarines). If one were to attempt to generalize the results of [80] beyond 2- and 3- to n-dimensions (as already done in [169] for just the theory and proofs), a modified version of the computational approach of [43], [55], [244] may be useful in this endeavor (and perhaps even for 2- and 3-D as well) since the iterative algorithm used is a contraction mapping with a provably geometric rate of convergence (but needing double precision for all matrices and vectors involved). The use of an iterative solution technique is not necessarily at odds with providing real-time answers and may be the simplest path to follow. A navigation application using an even easier criterion of ellipsoid containment in dimensions higher than 3-D is discussed in [10]. *Cross-sectional processing* [175] should also be useful as a technique to use for sensor fusion.

III. Perspectives on Unsented Filters, Particle Filters, and New Cramer-Rao Bounds for Radar

We are aware of the following approach that evolved (Nahi '69, Jaffer and Gupta '71, Hadidi and Schwartz '79, Monzingo '75, '81, Askar and Derin '84, and Tugnait and Haddad '75) to handle situations where there is data dropout or missing data but we will not dwell on it here (other than to point out its pitfall) because it sought to complicate the situation well beyond what was needed. The above seven references attempted to fit an inappropriate computational architecture that sought to force use of a constant uniform step-size and anticipated periodic measurement availability throughout the implementation [11], where any lack of measurement returns at an anticipated time-step k would be modeled using a scalar independent multiplicative random variable γ_k that takes two possible values, either 0 or 1, within the standard expression for the received measurement data:

$$z_k = \gamma_k h(x_k) + v_k. \tag{22}$$

[10] Observe that a solution to the well-known generalized eigenproblem $\lambda Ax = Bx$ [168, Sec. 7.7] is also a solution of the fundamental Eq. 12 of [80] since $\lambda Ax = Bx \Leftrightarrow [\lambda A - B]x = 0 \Rightarrow x^T[\lambda A - B]x = 0$. Use of *Choleski factorization* and the *symmetric QR algorithm* are offered in [168, Sec. 8.7.2] as a stable solution for the case of A, B being symmetric and A being positive definite, as is in fact the case for the matrices encountered in [80] and herein. Observe that [80] deduces overlap by focusing and dwelling on how pairs of the eigenvalues of non-symmetric $A^{-1}B$ behave. Symmetric matrices have all real eigenvalues and just a consideration of symmetric matrices apparently suffices in a complete test while non-symmetric matrices frequently have complex eigenvalues and lead to more structural considerations that "muddy the water" in deducing whether there is overlap.

[11] This architecture arises in academic simulations so the problem occurring was in not recognizing at the time how to properly transition from the construct to be used for INS navigation to the proper construct to be used for radar.

In the above expression, the missing measurements correspond to $\gamma_k = 0$ for only noise being received. When $\gamma_k = 1$, the desired signal is present in the measurements. The problem with this formulation is that no real structure is available for predicting the behavior of γ_k as a function of discrete-time index k and, because of its presence, even applications possessing linear systems and linear measurements become horribly nonlinear and intractable as an infinite dimensional nonlinear filtering problem [12] even though it originally appears to be finite dimensional and was hoped, at that time, to be a slight perturbation of a standard Kalman filter. Use of a more appropriate architecture that just processes measurements after they have been detected to be present by the received signal exceeding the prescribed mandatory detection threshold avoids these problems and is more straightforward to implement [174] (by not assuming or relying on having a constant step-size between measurements but just using the existing time-tag [13] of the last received measurements to propagate to the next available one). When handled this way, no measurement is ever missing but postponed by just having to wait for the next available one to arrive.

The above approach to estimation somewhat parallels what is attempted at first in the beginning of [9] for optimal Cramer-Rao Lower Bound calculation (but is subsequently dropped in [9] after it was realized that it would be too unwieldy to figure out beforehand which combination of times will actually exhibit radar returns received). A tractable approach is instead invoked later in [9] (as in [17], [18, Lemma 6.5, Ex. 7.2, Sec. 7.10]-[20, Sec. 6.2], [22]-[24], [53]) of using the known times at which radar returns were actually received before calculating the CRLB for that situation. This approach is quite acceptable since CRLB calculation is usually done a posteriori and super-imposed on performance graphs afterwards. Using the analogy of the Fisher Information Matrix in the “denominator” before matrix inversion flips it up so it may be directly compared for proximity to computed covariance evaluations obtained by Monte-Carlo sampling, this inverted denominator is used in the same role as Kalman filter covariance analysis for the linear case (where the post- and prior covariances are identical for stipulated times of sensor measurement receipt).

On-line covariances can be calculated a priori if one has a perfectly known linear model, knows the covariance of the noises, and knows the exact times at which the measurements are received without actually needing the measurement realizations themselves. Since radar RV target tracking is nonlinear, we need to know the actual measurement realizations in performing the EKF estimation in order to estimate the state, about which the linearization is to be performed at each time step. However, in evaluating CRLB’s given the parameter (being the state, which is perfectly known in simulations), there is no need to know the measurement realizations too but just the times at which they are received and the corresponding covariance of the noises at those times. This was ultimately done in [9] as it was done earlier in [17]-[24]; however, [9] can handle the case of the truth model having non-zero process noise based on the theoretical result of [25]. Other historical attempts to handle non-zero process noise in decentralized filtering have their own drawbacks [68]. The four different decentralized filtering architectures for possible sensor fusion developed by the SDI Panel 10+ years ago (by Dr. Oliver Drummond and also presented at his 1997 short course at SPIE) were all predicated (as he rigorously acknowledged) on there being zero process noise present, otherwise these structures are useless or detrimental in situations where process noise is present (so they are inappropriate to use for Indo-atmospheric tracking). Ref. [68] discusses a decentralized version of filter fusion where the presence of non-zero process noise covariance was not well accommodated because the same small fraction of Q, being $\frac{Q}{M}$, was apportioned to each of the M participating filters (thus causing each participating filter to perceive the system as being much more benign than actual and be untuned to the true underlying situation) and likely fail in its objective of adequately tracking for each, which then adversely affects the collated whole (since each of M subset filters will fail to track a single target system’s plant that is much noisier than each filter expects) the aggregate can not be much better and the computer burden is M times larger than would be the case if just one correctly modeled filter were used (thus this particular approach defies the reasons for seeking a decentralized solution in the first place).

Dr. Dana Sinno (Lincoln Laboratory of MIT) has investigated *self-organizing* networks of Kalman filter-based sensors (not unlike the electronic acoustic sensors dispensed into the jungles of Viet Nam in the late 1960’s and 1970’s in a failed attempt to monitor Viet Cong activity in the vicinity). Now the sensors have a degree of intelligence and an ability to hierarchically self-organize (like the 1970’s vintage **JTIDS RelNav** did for the U.S. Navy) and automatically turn-off to conserve power when noisy activity is absent. These smart sensors can be interrogated and the master sensor reports back results to a command center (which can be one of several to avoid a single point vulnerability). Other related activity along these lines is reported in [272]-[275], [288].

Only ideal exact initial conditions were presumed for each target model for each local Kalman-filter-model-based sensor, so results may be less encouraging when practical initialization is eventually invoked (a candidate being [223]). Recall that the local sensor models are differential equation based and apparently assume only constant velocity. Once ideal exact initial conditions are inserted in such a differential equations-based or difference equation-based model ($Q = 0$), the target’s trajectory is completely determined precisely without any measurements being available. Subsequently providing sensor measurements is just “icing on the cake” by improving the on-line computed covariance but the target locations are already known precisely without ambiguity. Such would not be the case if the targets were allowed to accelerate later after tracking had initiated as long as it wasn’t the same

[12] Exact solutions in nonlinear filtering are infinite dimensional, in general, although certain extremely simple finite dimensional special cases exist [192] (1979) and as extensions to those of Beněs (’81, ’85, ’87); as Daum (’86); and as Tam, Wong, and Yau [120] (and their many further subsequent generalizations [126]); also see Stafford (’84) and [51, Sec. 2.3], [190], [191, Chap. 5 & Appen.], [196], [197], [232, Chap. VII], [234].

[13] For perspective, National Instruments’ commercially available recent M-series©Data Acquisition devices all routinely provide such time-tags or time-stamps automatically. Military turnkey applications have done so for decades.

constant acceleration as provided in the initial conditions. Even the use of available (but noisy) Doppler measurements from the radar sensors degrade the accuracy available from using mere position measurements alone in this overly idealized experimental situation (as a first step). A real difficulty at present is how to handle multi-target tracking at the sensor level. It is likely that this aspect will only be handled at the level of the supervising Command Center(s) as master station(s). This was an **IR&D** project reported on at the 2004 MIT Lincoln Laboratory **ASAP Workshop**. Dr. Sinno is now performing follow-on activities under a DARPA contract at Lincoln Laboratory (2004-5) that is, hopefully, more realistic. The late Prof. George N. Saridis (RPI) had published a book on self-organizing systems [194]. The novel *MaxPlus* formulation, as reported in [249], [268], posses considerable potential for further revolutionary developments in estimation for networked systems described by Petri Nets.

Within [220], the IMM Filtering approach is again implemented only for targets described by linear systems, even under maneuvers. Strategic Reentry Vehicles (RV's) are targets known to be adequately modeled only as being nonlinear in both the dynamics and in the algebraic measurement model [218]. Ref. [220] also considers *Particle Filtering* (PF) and Probabilistic Data Association (PDA) approaches as well. As with almost all PF examples to date, the state space target dynamics model used within the calculations is planar and consists of only 4 states. The complexity of PF implementation as a CPU computational burden increases drastically as the state dimension increases. As already stated, practical Early Warning Radar target models must be at least six states, and must be nonlinear to realistically reflect the action of an inverse square gravity and the oblateness of the earth (effects that cannot be ignored in a realistic EWR application). Unfortunately, ref. [220] does not reflect Bit Error Rates (BER) or possible request for retransmission within the communications channels as a consequence of BER. Ref. [220] does not consider having adequate processor capability at each sensor site, as would be the case for EWR with two way communication links present to Colorado Spring's Cheyenne Mountain, and which is more compatible with methodology of decentralized filtering, reported on herein.

Particle Filtering utilizes numerous "mini-simulation" trials before each measurement incorporation step [242], [298]. That is a big computational burden and Gordon et all did not hide that fact in 1993 [7] when they came up with the original concept of *Particle Filtering*. They acknowledged that they could do many things in a better way to reduce the computational burden (selective sampling and re-sampling, invoking Metropolis sampling (1953)[301], Metropolis-Hasting's sampling method, invoking Metropolis-Gibbs' sampling method, etc.) and [7] predicted future reductions in computational burden. Sure enough, later versions of Particle Filtering, as obtained 9 years later by the same authors (and by others, such as Rudolph van der Merwe's *ReBEL* and Frederick Daum and Jim Huang [8]), made more efficient use of computations and the computer burden was significantly reduced. However, the CPU burden is still considerably larger than is likely tolerable for EWR. Fred Daum (Raytheon) compared the old and new versions of Particle Filtering (denoted as *Plain Vanilla* and as *Bells and Whistles*, respectively, as PF-PV and PF-BW) to verify the reduction and improvements and also offered theoretical results that serve as a bound on the likely computational burden of future improvements in Particle Filtering. Refs. [207], [208] have imaging applications of the PF's too!

At present, PF is too big a CPU burden to be practical (except [219]) and likely will remain so for the immediate future (cf., [255]). Larger state sizes, as needed in 6 state RVCC target tracking, would constitute a larger burden than quantified in Farina's paper [10] for an unrealistically benign situation of using only a 4 state filter for endo-atmospheric tracking, with target trajectory confined to a known plane [14] also containing the observing radar, and with ballistic coefficient (associated with atmospheric drag) being presumed completely known. Farina's Particle Filter was 440 times the computational burden of his EKF of the same state size (see Table 2). Because the particle size necessary to support a Particle Filter increases with the state size, realistic filter dimensions of seven states would likely cause the corresponding Particle Filter to be an even larger computational burden than 440 times the computational burden of the corresponding 7 state EKF [15]. We await and encourage further innovative constructions of proposal densities that should help concentrate the intermediate Monte-Carlo simulations [301] routinely present in PF to occur in more useful areas of state space so less of the simulation effort is wasted than as occurs now.

While the Unscented Kalman Filter (UKF) [1] is now portrayed in a better light in [2] [16] and in [4], [164], [277], the UKF is still a bigger computational burden but not as large as a standard "particle filter" (PF) [7], as aptly illustrated in [10] by comparing the performance of 4 different estimators: EKF, *Statistical Linearization* (KADET), UKF (Julier-Uhlmann),

[14]Elliptical motion is exhibited by satellites and by reentry vehicles. For both, object position and velocity are governed by the nonlinear dynamics of body motion in a central force inverse squared gravity field. An accurate model should include considerations of the second harmonic J_2 for realism to account for the oblateness of the earth (and its presence induces two more characteristic motions known as the *regression of nodes* and the *rotation of Apsides* [296]).

[15]CPU times or flop counts of EKF's implemented on sequential Von Neumann machines go as a multiple of n^3.

[16]In 1997 during **HAVE GOLD**, TRW was funded to implement the Unscented Filter a.k.a. the Oxford Filter, but failed. Our apprehension of UKF is because of: presence of an unexplained factor (or unconstrained non-integer free parameter, possibly positive, negative, or time-varying at the whim of the analyst) that can serve as an expanding or contracting twiddle factor in the denominator of the gain expression that is consequentially inherited by the covariance equations; numerical comparison in [1] of UKF vs. EKF performance appear contrived since real EKF practitioners would either take more frequent measurement fixes or better pose the target model to take into account its anticipated motion about a circular track (thus avoiding literally going off on tangents); lack of usual local *Lipschitz* assumption that would indicate awareness of minimum conditions needed for solution to exist for the underlying stochastic differential equation model (but do invoke conditions that are impossible to check beforehand e.g., [1, Eq. 2] since probability measure for $x(k)$ is unknown); unconventional use of calculated covariance to account for nonlinear measurement equation and associated unconventional assumption of mean being zero and unconventional proposed handling if mean is not zero (saying it can be shifted, but mean is in fact unknown so one can not know beforehand how much it should be shifted by so user is stymied in trying to proceed [1, Sec. 4]); UKF also utilizes "mini-simulation" trials before each measurement incorporation step.

TABLE II
RELATIVE CPU BURDEN OF 4 DIFFERENT ALGORITHMS [10]

EKF	KADET	UKF	PF (N=25,000)
1	300	3	440

and PF (with n=25,000) for an over simplified version of endo-atmospheric radar target tracking (where the nature of the over simplifications are examined more closely in Sec. 4). Taking the computational burden of the EKF to be unity, the relative computational burden of the four filters was tallied in [10], as summarized here in Table 2.

The study in [269] stacks up benefits and drawbacks to conclude that both EKF and UKF performed well for integration of MEMS-based IMU with GPS, with and without GPS blockage. However, Unscented Kalman Filter (UKF) was deemed superior in [269] for seamlessly handling situations where large initial attitude errors are present without needing to initiate special in-motion alignment procedures. An observation here is that this post-processing analysis study neglected to include a consideration of total operations counts incurred or the ability of a particular algorithm to keep up with processing demands in real-time navigation applications, where timeliness of computed results should be a primary consideration. This aspect is where use of the Unscented Kalman Filter is less satisfying and where use of an Extended Kalman Filter (EKF) wins the contest hands down. Our last comment is based on our prior experience in both INS/GPS and in EKF. Another study [271] directly compares the performance of the UKF to that of the EKF to also favor UKF use as being more accurate (but also apparently failed to emphasize the important hard real-time constraints present in the application).

With the passing of time, further refinements have been made and approximate approaches have been developed [253] that trade-off computational complexity against optimality (and associated accuracy) of the above algorithms, as has been quantified across the board for the same identical example application problem. Frederick Daum's (Raytheon) March 2003 IEEE Aerospace Conference paper on Particle Filtering [8] demonstrates that, although the Particle Filter is easier to code, in some simple situations, the Particle Filter can be 10^5 to 10^8 more computationally burdensome than an Extended Kalman Filter for comparable accuracy. This is generally consistent with what Farina et al reported. Ref. [177] is an excellent, well-written yet concise book on the many aspects of radar tracking, as it affects algorithm selection; but finally admits PF drawbacks in the *Epilogue* [177, p. 287].

The benefits of possessing a better (second order) approximation to any nonlinearities present in the system or measurement model, that is touted for PF and UKF (at the expense of incurring multitudinous "mini-simulations" at each measurement update step of the filter), would be an important consideration and lucrative aspect if the dynamics and/or measurement models were not well known. This is definitely not the case for Exo- and Indo-atmospheric radar tracking. A 2^{nd} order filter variant or extension of an EKF (also known as a Gaussian Filter) achieves a second order accuracy in its approximation of these well-known and carefully modeled and documented nonlinearities (e.g., [59, p. 33]) by retaining the first three terms in the Taylor series representation of each using the corresponding Jacobian (1^{st} derivatives) and Hessian (2^{nd} derivatives), which need be computed only once as an off-line a priori analytic exercise, as discussed further in Sec. 9.

The Interactive Multiple Model (IMM) bank-of-Kalman-filters approach arose and is apparently supplanting the 1965 approach of Magill, which is architecturally similar but lacks IMM sojourn times (of an associated underlying Markov Chain) as the contrivance that keeps the several filter options alive as viable alternative filter models being continually actively entertained as possibilities as different operating regimes of the system are encountered. Such IMM approaches are being considered for tracking maneuvering targets and missiles that are boosting but nonlinear IMM probability calculations are currently impossible to compute in real-time [222] (also see [265]). Ref. [156] combines both IMM and PF for Multi-target tracking. Concerns regarding IMM may be found in [265], [58, Sec. 12] along with a list of 6 other cautions in Sec. 9 herein relating to the vagaries of approximate solutions to nonlinear filtering.

IV. LACK OF MISSILE DEFENSE REALISM IN SOME TRACKING ACCURACY EVALUATION SCENARIOS

Further elucidating our concern regarding lack of realism by Farina et al's reentry model by its being completely planar in [10], central forces in 3-D give rise to trajectories that are confined entirely to be within a specific plane (known, historically, as the osculating plane). From a *physical mechanics* course, one learns about *central* force motions and associated properties. Such studies reveal that, for a central force field (like inverse squared gravity), the following cross-product $r \times \dot{r}$, where r is the position vector from the origin of a coordinate system erected at the center of gravity of the earth as focus to the location of the projectile, defines the normal to the plane in which the motion of the projectile is confined. In actual radar applications, the ideal behavior is not precisely obtainable because of the *range-Doppler ambiguity* encountered, as associated with use of practical radar measurements; so there are slight errors present between measured range and its associated range-rate to some degree which, further, corrupts the accuracy of the effective estimates $\hat{r}$ and $\hat{\dot{r}}$ that together degrade the estimate of the normal to the *osculating plane* to which the projectile is confined, to the degree of departure from the ideal as indicated from the calculation of $\tilde{r} \times \tilde{\dot{r}}$, where the individual contributing errors are $\tilde{r} \triangleq \hat{r} - r$ and $\tilde{\dot{r}} \triangleq \hat{\dot{r}} - \dot{r}$. The effect of *regression of nodes* and *rotation of apsides* (mentioned earlier) aggravates the problem of instantaneously estimating the correct plane of projectile confinement even more since, instead of being merely constant and fixed, the osculating plane now moves due to the oblateness of the earth. While Farina's tracker is being treated as a problem that is entirely planar in [10], the real world problem in Missile Defense is to actually figure out what plane the trajectory is situated in and how it is posed. By ignoring these real world effects, Ref. [10], as a consequence reaps greater accuracy than is likely in practice, where four other out-of-plane errors arise (that are ignored in [10]). In like manner, Paul Zarchan (Lincoln Laboratory) et al used this same contrivance in evaluations presented at Colorado Springs at 1997 *AIAA/BMDO Symposium and Workshop* for RV target tracking via radar. Zarchan and Jesionowski [313] used a 5 state Extended Kalman filter (but also used inverse square gravity instead of Farina et al's constant gravity assumption) so Zarchan et al obtained even better results than [10] (because inverse square law gravity, although nonlinear, sets up a gravity gradient that varies with altitude and observed target behavior and better pin-points actual altitude and associated location as a consequence). Results were better in both these simulations than can be obtained in practice since both assume the plane of motion was already precisely known (and consequently optimistically treat the component of out-of-plane errors as being nonexistent and zero in the tally of total error incurred). Part of the real problem is to actually deduce in what plane the target is traveling. In some cases such as in benign pre-planned test shots from Vandenberg AFB in CA toward the Kwajalein Atoll [17] in the Marshall Islands using our own cooperative target Reentry Vehicles (RVs), one may know the launch point precisely as well as the aim point and our own missiles may have the reentry angle completely known to us for tracking purposes (but for actual missile defense, the defender's tracking radar usually only has a view of the target during a portion of midcourse through reentry and we know that anticipated launch points can be varied via use of wheeled or rail vehicles or submerged submarines and that a sophisticated enemy can also suddenly vary several parameters relating to reentry drag during end-game). Farina et al also assume that the observing stationary ground-based radar is at the launch point (and in the plane of the target trajectory), as a considerable advantage of having almost perfect initial conditions for the target tracking algorithms almost from the start, which is an unrealistic assumption for Missile Defense [18]. In reality, a lack of sufficiently accurate initial conditions is a big handicap that arises in actual target tracking.

Moreover, when atmospheric drag enters into the picture, as it did for both [10] and Zarchan and Jesionowski's Reentry tracking examples [313], the problem is no longer an exclusively central force problem (guaranteed to be planar). Any tilt of the reentry body forces the trajectory out of the expected osculating plane, which both simulation studies treat as being perfectly known but didn't list among their assumptions. The much more sophisticated follow-up investigation of [151] does not have any of the questionable aspects mentioned above and the trajectory analysis is first rate; however, there are still some concerns:

1) An $\alpha - \beta$ filter is historically well-known to be merely a special case of a Kalman filter under the *assumption that velocity* [19] *is constant* and the filter is run to *steady-state* to get the corresponding constant gains [154, p. 23]; why draw a big distinction now after three decades of *Moore's Law* being in effect [281] to guarantee ample computer capacity and speed now being available? Although [151, p. 621] is correct in its tallies, nickel and dime operation count bookkeeping comparisons of these steady-state-only algorithms vs. use of a Kalman filter (which can track through the transient regime) are less pressing than they once were 30 years ago, when computing resources were scarce. Most EWR applications nowadays use parallel processors and have blazingly fast operating speeds. However, such tallies for a Von Neumann sequential machine are still a useful comparison to check as parallel implementations are pursued that should be faster.
2) The derivation of $\alpha - \beta$ filter implementation equations, as spelled out in detail in [151], have apparently been obtained previously by A. W. Bridgewater, as reported in the proceedings of an earlier AGARD Conference [152, p. 38], [153] within the context of automatic track initiation and, in like manner, using decoupled components too.
3) It is stated in two places within [151, ff Eq. 31, ff Eq. 38] that the elements of the appropriate Q (expected to be the compensating fictitious process noise covariance) comes from Eq. 48, yet [151, Eq. 49] is the expression for the initial covariance P_o to be used for filter initialization and is apparently not for Q. (Perhaps it was a typo that occured twice?)
4) Radar at sensor location 1, depicted in Fig. 2 of [151], is essentially broadside of the target trajectory (an extremely favorable position for tracking the target but not a likely position for land-based EWR already deployed at fixed known locations). Will resulting target tracking accuracy evaluations be representative of likely EWR performance (or, instead, be representative of only ideal behavior in the situation of best case geometry like this)?
5) Radar at sensor location 2, depicted in Fig. 2 of [151], is still essentially also broadside but now extremely close to the target and will naturally reap the benefits of an extremely large SNR detection and tracking signature. (Perhaps this is a subtle advocate for sea-based EWR by "stacking the deck" in this manner, without indicating this *assumption* in [151]).
6) The good aggregate performance observed (i.e., from averages of 100 Monte-Carlo runs) for three of the four older more conventional historical algorithms under consideration in [151] being close together in accuracy may have been the consequence of the close proximity of

[17] The Tradex radar used for tracking these RV test shots is located there at KREMS (along with Altair, Alcor, and MMW). The L-band Tradex radar has MTT for up to 63 simultaneous targets appearing within the same mechanically scanned 0.61^o, 6 dB_m beamwidth pencil-beam of the 25.6 meter diameter antenna, but has a 600 meter blind zone behind the primary target cluster grouping [150]. Actual EWR usually uses phased arrays and electronic scanning.

[18] Where initial conditions for each target must be deduced from detection and confirmation waveforms, available pulse patterns [295], or from other supplementary information [54].

[19] Heroic and novel compensation techniques were used to compensate for acceleration and velocity not being constant. A recent improvement to mere $\alpha - \beta$ is [305].

the radar locations relative to the target and almost ideal constant periodic measurement availability (exclusively at rates of, first, every 0.25 sec., then later at rates of every 4 sec.) utilized (initially without any data dropout gaps that would otherwise be present in practice with realistic radar schedulers that manage their finite antenna beam and signal processing resources between threat target tracking and surveillance fence monitoring in also performing the background bookkeeping to properly account for orbital debris and existing satellites in the 9,000+ item Space Track Catalogue).

7) To be fair, data measurement gaps were later introduced in [151] but were extremely regular by periodically missing a whole batch of measurements (at the aforementioned rates) for 20 seconds then receiving all for the next 20 seconds and continuing to alternate in that fashion. Another situation in [151] looked at just one data dropout segment of 50 seconds duration arising 250 secs. after tracking had commenced and already settled out. Targets were non-maneuvering and so coasting with the inertia of the previous target history still gives pretty good target tracking accuracy as merely an extrapolation step since atmospheric drag likely had not kicked in yet as being significant. (Ideally, accompanying algorithm self-assessment on-line covariances should have been large when drop outs occurred but no associated covariances were reported in [151].)

One could question why it appears that the deck is always stacked in their favor in [10], [151] when they evaluate the performance of their own algorithms? Was it merely a coincidence?

A varying ballistic coefficient under enemy control but unknown to the defender can be a very challenging problem to track. George Souris (AFIT, retired) had an interesting approach for handling the radar tracking of an RV with unknown ballistic coefficient [26] by treating the ballistic coefficient within the theory of Interval Matrices, where all entries of the matrix were explicitly known except the entry associated with the ballistic coefficient. This model matches the true situation in EWR for indo-atmospheric targets. This unknown entry was known to be confined to within a reasonable range or "interval", hence the name. The theory of Interval matrices has recently had some interesting and useful results [27] that make it even more alluring.

V. PRECEDENTS IN CRLB EVALUATIONS FOR EXO-ATMOSPHERIC TARGETS

Prescribed Measures of Effectiveness (MOE) for EWR applications currently exist as a gauge of tracking algorithm effectiveness, including widespread use of 97% Spherical Error Probable (SEP). However, there is a problem with this highly touted MOE, as demonstrated in Sec. 5.1. We offer the Cramer-Rao Lower Bound (CRLB) as a less ambiguous MOE and discuss its theoretical basis in Secs. 5.2-5.4 and provide a representative numerical example for EWR in Sec. 5.5.

A. Why use CRLBs for evaluating EWR Target Tracking Efficacy?

Simple transparent examples of 97%SEP behavior (or, more precisely, 97%CEP behavior which identically parallels in one lower dimension for simplicity the 97%SEP situation) will be presented here. Initial Tracker performance evaluations frequently consist of 250 trials as a goal. While 250 is a fairly large number and is generally obtained with fairly high computational expense incurred in obtaining the requisite realistic Monte-Carlo simulation runs of a particular tracking estimation algorithm under scrutiny, it still does not yield infallible results. To illustrate this claim, please consider the four different histogram diagrams depicted in Fig. 5 (a), (b), (c), and (d) for a uniform histogram . bucket size (not mandatory).

Consider that for x_1 and x_2 being zero mean independent Gaussian random variables with the same magnitude variance [20], the following miss distance, as the square root of the sum of the squares of the constituent position components: $y = \sqrt{x_1^2 + x_2^2}$, is well known to be Rayleigh distributed [304, p. 195]. For 250 trials, we have that from a true Rayleigh distribution, $\frac{y}{250} = 0.97 \rightarrow y = 242.5$ so by having the results of 242.5 trials to the left of a particular value on the abscissa corresponds to the critical value being sought (deemed to be 97%CEP for 2-D, as depicted here in Fig. 5 (a), (b), (c), (d), and (e). The same results would be obtained using the alternate cumulative distribution form of these same histograms. (The corresponding 3-D miss distance corresponding to 97%SEP having three independent Gaussian constituents (each component assumed to have an identical variance) would have a Maxwell distribution [304, Ex. 8-5, p. 273].)

These simulations were simply obtained from the "Histogram Generator" within the demonstration programs for the *Statistics Toolbox* provided by The MathWorks that may be used within MatLab©. The four runs depicted above ware for a Rayleigh distribution with the B parameter being 4 (constrained to be between 1 and 6). Notice that for each of these four trials of 250 samples each, a different value of 97%CEP was obtained of 11, 12, 13, and 14 units for, respectively, cases a, b, c, and d. A less ambiguous MOE for gauging the efficacy of target tracking is discussed next.

B. Review of analytical basis and procedure for evaluating CRLBs for EWR target tracking

Under the standard assumption that the estimator is unbiased [21], then the familiar form of Cramer Rao inequality

[20] In actuality, the variance would likely be different for each constituent component but still the miss distance is just as obviously non-Gaussian even when each component is Gaussian. The main point is that the miss distance is blatantly non-Gaussian in reality as well as for the ideal of all component variances being the same.

[21] The bias referred to here is inherent to a particular estimator and is generally not directly related to any underlying fundamental biases arising for reasons other than the structure of the estimator being employed within a particular application. The techniques for removing biasedness from estimators such as that by D. Lerro and Y. Bar-Shalom (1993) have been for situations where the system Dynamics are linear [287], [311], [312] (unlike the case for EWR).

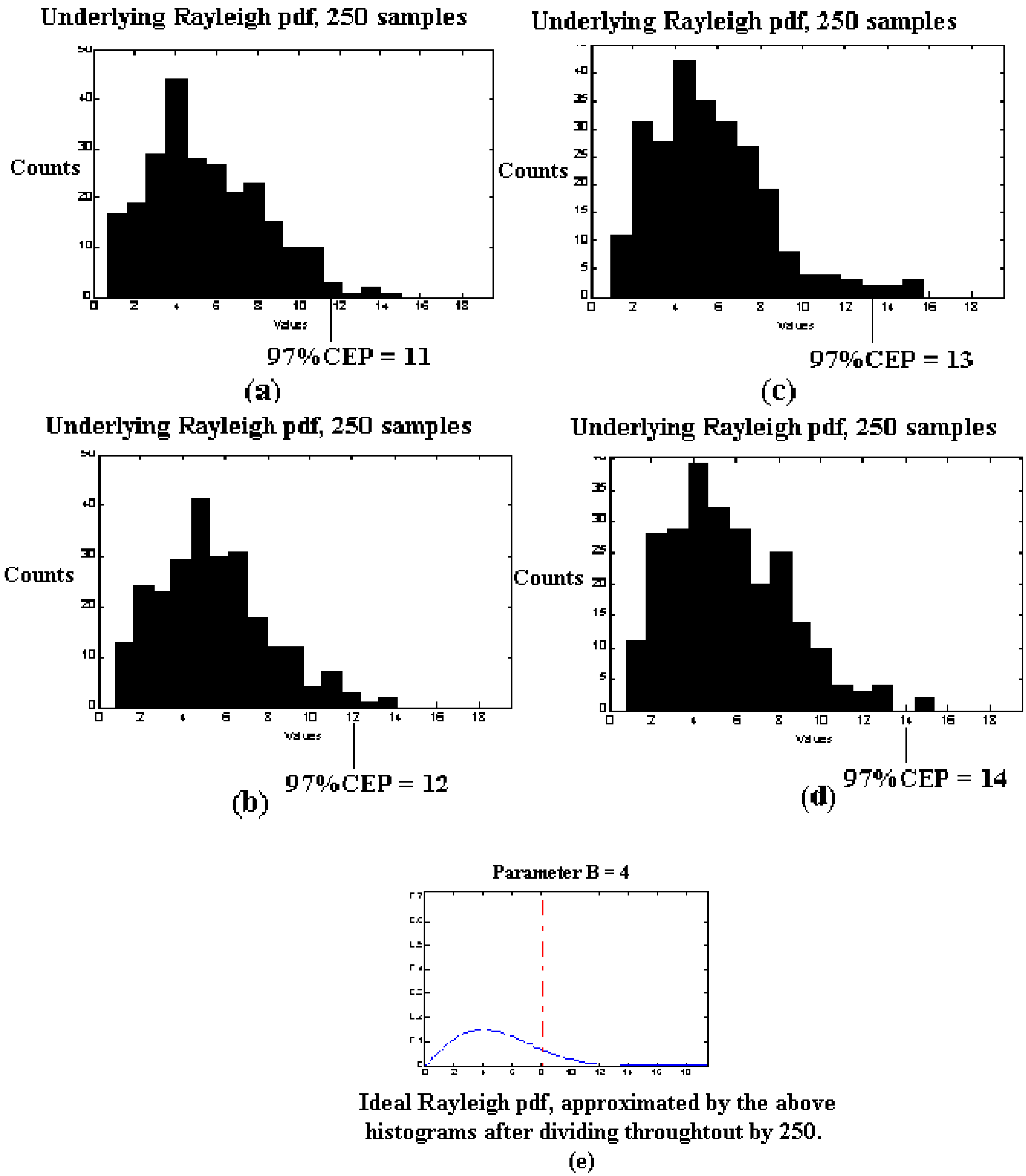

Fig. 4. 97%CEP Results for 4 separate Histogram samples (N= 250) having a common underlying Rayleigh Probability Density Function (pdf)

encountered or invoked most frequently is:

$$E[(x-\hat{x})(x-\hat{x})^T|x] \geq [-E\{(\frac{\partial}{\partial x})^T(\frac{\partial}{\partial x})\ln\{p(z|x)\}\}]^{-1} \equiv \left(1+\frac{\partial B}{\partial x}\right)^T \mathcal{I}^{-1}\left(1+\frac{\partial B}{\partial x}\right) \quad (23)$$

where the inequality here for these matrices is interpreted in the matrix positive semi-definite sense (i.e., $A \geq B \Leftrightarrow A - B \geq 0$) and $\mathcal{I}$ is the *Fisher Information Matrix*. Please see references cited in [21] for details. It is this form [22] (under the widely invoked assumption that the estimator bias is either non-existent or negligible) that has a RHS that is independent of the particular estimator being used and that may be compared to a wide variety of distinctly different estimators as a single relative gauge throughout. The CRLB methodology is used here to gauge the quality of filter performance in the tracking task.

The above Cramer-Rao inequality arises in seeking to estimate an unknown parameter x using any estimator $\hat{x}$ and the measurement $z(t)$, referenced above and available from the sensor as a time record, must be a non-trivial function of the unknown parameter x as

$$z(t) = h(x, t, v(t)) \,. \quad (24)$$

In the above, $p(x|z)$ is the conditional probability density function (pdf) of x given all the measurements z, and $v(t)$ is the measurement noise. In exo-atmospheric target tracking, x is deterministic and satisfies a known nonlinear ordinary differential equation and v is additive Gaussian white noise of known covariance intensity, hence $p(z|x)$ is known.

Although other bounds exist, like that of Barankin, the CRLB was selected for use as the familiar bound most appropriate for EWR application because it matches the situation and is tractable. A high-level overview of the CRLB methodology and its benefits and limitations may be found in [21] while it is specialized specifically for EWR exo-atmospheric tracking in [22]-[24].

The CRLB being achieved means that the error of estimation term on the LHS of Eq. 29 touches the CRLB term on the RHS by satisfying the indicated inequality as an exact equality. For EWR target tracking, the lower bound should not generally be achievable (hence this CRLB is **not** expected to **exactly** match the average sampled tracking error variance compiled from N Monte-Carlo trials).

This CRLB was derived by adapting a time-varying radar SNR to realistically correspond to fluctuating PRF and other underlying signal processing as an enhancement of the fundamental methodology that evolved in [17]-[20], as tailored to this EW radar application using the conventions laid out in [22]-[24]. The procedure of [22]-[24], [53] already considered $P_d < 1$ since it included explicit consideration of the detection threshold settings and, moreover, used measurement reception times that correspond to the time-tags for when measurements were actually received (so these CRLBs are a posteriori bounds). While [9] initially tries to tackle a more general case of a priori bounds, it found that approach to be intractable and so [9] then merely resorts to using the structure of CRLBs in [25] for handling process noise that is not zero (as occurs in Indo-atmospheric reentry tracking and not in exo-atmospheric tracking, where $Q = 0$). This is the big distinction between the CRLB approach of [9], [25] and that of [22]-[24], [53], where the latter constitute a much lower CPU burden.

[22]The summarizing notation $\mathcal{I}$ appearing on the Right Hand Side (RHS) in Eq. 23 is known as the Information matrix prior to matrix inversion, after which the entire expression (after $\frac{\partial B}{\partial x} \to 0$) is the so-called or so-designated Cramer-Rao Lower Bound (CRLB), which can be numerically evaluated.

C. Insights into when and why CRLBs sometimes appear to have weird behavior

There is frequently a small initial time segment in the beginning of an estimation error plot when an estimator's covariance lies below the CRLB (as it should, considering the approximations that are usually invoked up to that point, as will be explained) before switching to the usual situation of the CRLB lying below. Sometimes the initial values are so large and far off that, by the automatically adjusted vertical scale inherent in many plot packages, this initial switch appears to be such a proportionately small segment of the figure that it does not raise suspicion or concern enough to be explained to an audience of readers. The same type of thing occurred for each of Farina's four estimators in [10]. To the present author, this is a mark of honesty in the preparation of the results (but impedes a presentation somewhat in a situation where the speaker has to stop and explain why an apparent bewildering situation occurs of the direction of the expected inequality flipping around and the expected lower bound being above the sampled σ). An explanation was not given in [10] nor has it been given anywhere else to this author's knowledge so we will do so here now.

The explanation is because of that numerator factor $\left(1+\frac{\partial B}{\partial x}\right)^2$ in Eq. 29 that is needed for CRLB to be below **all** the time, where B is the bias in the estimator. Since we don't usually have B explicitly available and even when we do, its sensitivity to the parameter (in this case the state vector) being estimated needs to be evaluated as the indicated derivative (which is usually not conveniently tractable) so it is usually ignored entirely since it can't be evaluated anyway and we focus instead on the denominator term (which is the inverse of the Fisher Information matrix), which we can evaluate numerically. The numerator term can be a magnifier or a minimizer, depending on whether it is greater or less than 1 and it changes with time. Since estimators frequently proceed to have a steady-state bias (where $\frac{\partial B}{\partial x}$ becomes zero), the exact CRLB expression (numerator and denominator) eventually converges to the approximate CRLB expression (involving denominator alone). Since we frequently have explicit access to only the denominator, we usually use just that and wait past the initial transient until it is appropriate to compare against because only then does the right hand side CRLB$\approx \mathcal{I}^{-1}$, where it is reminded that $\mathcal{I}$ here is the *Fisher Information Matrix* (c.f., [179]).

D. Analytical derivation of the CRLB for target tracking situations devoid of process noise

Ballistic trajectories that are exclusively exo-atmospheric correspond to targets with no process noise (i. e., $Q =$

0). The CRLB that is treated here goes beyond just using the historically familiar per pulse CRLB angle measurement error [23]of Eq. 41 (in [22]-[24], [53]) since our CRLB goes further to additionally utilize:

1) information provided by the target dynamics model over time in an inverse square gravity field,
2) the initial (starting) covariance $P(0)$ of the tracking filter as handed-over [24],
3) the structure of the radar as a measurement sensor/device having additive Gaussian measurement noise with parameters including:
   (3a) explicit use of the radar range uncertainty due to resolution size of the range gates, the monopulse SNR time-record with its adaptive step size (as a consequence of a
   (3b) realistically varying PRF) as it affects the corresponding angle uncertainty.

However, one CRLB version used the SNR records simulated by TD/SAT, as Government Furnished Equipment (GFE), and each sample function realization interpolated to common times throughout and then averaged (by Dan Pulido, General Dynamics) to obtain SNR values at designated periodic times, thus providing smooth CRLBs as an envelope for comparison to estimator performance at arbitrary step times.

For an exo-atmospheric EWR tracking application, which has additive Gaussian [25] white measurement noise $v(t)$, the radar measurements of Eq. 24 has this further benign and accommodating structure to be exploited:

$$z(t) = h(x,t) + v(t), \tag{25}$$

and since the equation for the system evolution is essentially deterministic (with $Q_c = 0$), then the pdf's of interest here (to be used in numerically evaluating the CR lower bound of Eq. 23) are of the form:

$$p(z|x) = \frac{e^{-\frac{1}{2}(z-h(x))^T R^{-1}(z-h(x))}}{(2\pi)^{n/2}|R|^{-\frac{1}{2}}}\ . \tag{26}$$

Now taking natural logarithms on both sides of the above pdf yields:

$$\begin{aligned}&\ln\{p(z|x)\}\\ &= -\tfrac{1}{2}(z-h(x))^T R^{-1}(z-h(x)) - \ln(2\pi)^{n/2}|R|^{-\frac{1}{2}}\end{aligned} \tag{27}$$

which upon taking the gradient is:

$$(\frac{\partial}{\partial x})^T \ln\{p(z|x)\} = \frac{\partial^T h(x)}{\partial x} R^{-1}(z-h(x)). \tag{28}$$

When the above expression is post-multiplied by its transpose and expectation taken throughout, the result is:

$$\begin{aligned}&E[(\tfrac{\partial}{\partial x})^T \ln\{p(z|x)\}\tfrac{\partial}{\partial x}\ln\{p(z|x)\}] =\\ &\tfrac{\partial^T h(x)}{\partial x} R^{-1}\overbrace{E[(z-h(x))(z-h(x))^T]}^{R} R^{-1}\tfrac{\partial h(x)}{\partial x}\\ &= \tfrac{\partial^T h(x)}{\partial x} R^{-1}\tfrac{\partial h(x)}{\partial x}.\end{aligned} \tag{29}$$

Finally, over corresponding discrete-time steps (not necessarily uniform in step size), the total pdf of the whole collection of independent (white) measurements is the product of each individual measurement pdf of the form of Eq. 26 as $p(z_1|x(0))p(z_2|x(0))p(z_3|x(0))\cdots p(z_k|x(0))$, where each pdf for each constituent measurement here focuses on or is conditioned on the initial condition for the deterministic system equation. Once the initial condition $x(0)$ is known with confidence, then the time evolution of the deterministic system is completely determined (as a consequence of initial condition observability). The corresponding information matrix for each of these measurement time points is of the form of Eq. 29 so the aggregate is of the form [26]:

$$\begin{aligned}&\mathcal{I}(k,0) =\\ &\textstyle\sum_{j=1}^{k} \Phi^{-T}(k,j)\frac{\partial^T h(x)}{\partial x}|_j R^{-1}(j,j)\frac{\partial h(x)}{\partial x}|_j \Phi^{-1}(k,j),\end{aligned} \tag{30}$$

for $k \geq j$, where the transition matrix $\Phi^{-1}(k,j) \stackrel{\triangle}{=} [\Phi(k,j)]^{-1} = \Phi(j,k)$ and, likewise, corresponds to an evaluation of the system matrix linearized about the true state. Now, when there is a finite initial covariance being utilized by the estimator as tracking commences, then there is an additional term [27] that should appear in the above Information matrix to properly reflect this situation, as depicted as the first term on the RHS here:

$$\begin{aligned}&\mathcal{I}(k,0) = \Phi^{-T}(k,0)P^{-1}(0)\Phi^{-1}(k,0)\\ &+\textstyle\sum_{j=1}^{k} \Phi^{-T}(k,j)\frac{\partial^T h(x)}{\partial x}|_j R^{-1}(j,j)\frac{\partial h(x)}{\partial x}|_j \Phi^{-1}(k,j),\end{aligned} \tag{31}$$

In either the case of Eq. 30 or Eq. 31 holding, the Information matrix can be interpreted or formulated as evolving recursively with each received measurement arrival time as:

$$\begin{aligned}&\mathcal{I}(k,0) = \Phi^{-T}(k,j)\mathcal{I}(j,0)\Phi^{-1}(k,j)\\ &+\tfrac{\partial^T h(x)}{\partial x}|_k R^{-1}(k,k)\tfrac{\partial h(x)}{\partial x}|_k\end{aligned} \tag{32}$$

and, as such, may be implemented within software as merely a loop (but by observing all the constraints and coordinate conventions, where $\frac{\partial f}{\partial x}$ is evaluated within the ECI frame and $\frac{\partial h}{\partial x}$ is evaluated in the (E,N,U) frame [28] with corresponding

[23]Receive sum-pattern beamwidth: $\theta_3 = 3\ dB_m$.

[24]Using a standard hand-over covariance of $(100km)^2$ for all three components of the position block and $(100m/sec)^2$ for all three components of the velocity block. Physically, this should originate and be communicated from Space Based InfraRed Satellites (SBIRS) for National Missile Defense (NMD) [54], [299]. The Levenberg-Marquardt method that has been advocated for use as cutting edge statistical curve fitting for SBIRS was ostensibly developed earlier by a numerical analyst at Dupont Laboratory in the 1960s. Unfortunately, although the source code may be found in [303, pp. 197-209], the reference cited there for Levenberg-Marquardt does not pertain to this particular algorithm at all. Suspecting a slight mix-up, we searched further for it in other publications by the same Dupont researcher that appeared at around the same time but to no avail. The Levenberg-Marquardt method probably should not be used for EWR without an explicit rationale being available.

[25]Gaussianity arises as a result of the *Central Limit Theorem* (CLT), which with weakened hypothesis, no longer requires that contributing constituents be independent and identically distributed (iid) and hypotheses have likewise been weakened to be more easily met so that conclusions can now be invoked from the *Law of Large Numbers* [84].

[26]After taking the natural logarithm of the aggregate pdf, the exponents in the Gaussian distribution correspond to the indicated sum, after performing a gradient and taking expectations, as illustrated in detail above in Eqs. 27 to 29 for just a single measurement for clarity.

[27]An additional "fictitious measurement" was called for in [17, following Eq. 4] as being needed to avoid encountering numerical difficulties but use of $P^{-1}(0)$ as suggested here appears to suffice as a remedy that arises naturally.

[28]A representation in sine space, centered within the antenna array, is recommended for consistency with EWR.

translation offset to the location of the tracking radar) [29]. We have particular interest in the total position error and the corresponding total velocity error to determine how well we are actually doing in tracking a target complex. To this end, we must rigorously contort the inequality of Eq. 23 to a form that we can use. This is accomplished by properly applying matrix operations that yield the expressions that we seek [30] as:

$$\begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \sigma_{13}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \sigma_{23}^2 \\ \sigma_{31}^2 & \sigma_{32}^2 & \sigma_{33}^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \cdot$$
$$E[(x_t(t) - \hat{x}(t))(x_t(t) - \hat{x}(t))^T | \mathcal{Z}(t)] \cdot$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T \geq \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \mathcal{I}^{-1}$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} crlb_{11} & crlb_{12} & crlb_{13} \\ crlb_{21} & crlb_{22} & crlb_{23} \\ crlb_{31} & crlb_{32} & crlb_{33} \end{bmatrix} \quad (33)$$

and

$$\begin{bmatrix} \sigma_{44}^2 & \sigma_{45}^2 & \sigma_{46}^2 \\ \sigma_{54}^2 & \sigma_{55}^2 & \sigma_{56}^2 \\ \sigma_{64}^2 & \sigma_{65}^2 & \sigma_{66}^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot$$
$$E[(x_t(t) - \hat{x}(t))(x_t(t) - \hat{x}(t))^T | \mathcal{Z}(t)] \cdot$$
$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T \geq \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathcal{I}^{-1}$$
$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} crlb_{44} & crlb_{45} & crlb_{46} \\ crlb_{54} & crlb_{55} & crlb_{56} \\ crlb_{64} & crlb_{65} & crlb_{66} \end{bmatrix}, \quad (34)$$

and then by taking the trace of a matrix throughout [31], respectively, yields **radial position error variance**:

$$\sigma_{position}^2 = \sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2 = \mathbf{tr} \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \sigma_{13}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \sigma_{23}^2 \\ \sigma_{31}^2 & \sigma_{32}^2 & \sigma_{33}^2 \end{bmatrix} \geq$$
$$\mathbf{tr} \begin{bmatrix} crlb_{11} & crlb_{12} & crlb_{13} \\ crlb_{21} & crlb_{22} & crlb_{23} \\ crlb_{31} & crlb_{32} & crlb_{33} \end{bmatrix} = crlb_{11} + crlb_{22} + crlb_{33} \quad (35)$$

and **total velocity error variance**:

$$\sigma_{velocity}^2 = \sigma_{44}^2 + \sigma_{55}^2 + \sigma_{66}^2 = \mathbf{tr} \begin{bmatrix} \sigma_{44}^2 & \sigma_{45}^2 & \sigma_{46}^2 \\ \sigma_{54}^2 & \sigma_{55}^2 & \sigma_{56}^2 \\ \sigma_{64}^2 & \sigma_{65}^2 & \sigma_{66}^2 \end{bmatrix} \geq$$
$$\mathbf{tr} \begin{bmatrix} crlb_{44} & crlb_{45} & crlb_{46} \\ crlb_{54} & crlb_{55} & crlb_{56} \\ crlb_{64} & crlb_{65} & crlb_{66} \end{bmatrix} = crlb_{44} + crlb_{55} + crlb_{66}, \quad (36)$$

and, finally, by taking squareroots throughout [32], respectively, yields:

$$\sigma_{position} = \sqrt{\sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2} \geq \sqrt{crlb_{11} + crlb_{22} + crlb_{33}} \triangleq \mathrm{CRLB}_{position} \quad (37)$$

and

$$\sigma_{velocity} = \sqrt{\sigma_{44}^2 + \sigma_{55}^2 + \sigma_{66}^2} \geq \sqrt{crlb_{44} + crlb_{55} + crlb_{66}} \triangleq \mathrm{CRLB}_{velocity}. \quad (38)$$

Please notice in the above that we do not decouple position and velocity states but merely project both of the 6 x 6 matrices of Eq. 32, respectively, into the position subspace (as Eqs. 33, 35, 37) and into the velocity subspace (as Eqs. 34, 36, 38) for viewing in a plotter display. These instantaneous inequalities are now the theoretically justified comparisons that we invoke in monitoring performance of a target tracking algorithm as a function of time.

E. Assessing BASELINE Performance: comparing existing standard EKF to the computed CRLB

We illustrated the CRLB calculations relative to ensemble sampled Monte-Carlo results for the BMEWS radar: Thule [33] tracking an RV on a ballistic trajectory (post cut-off) having the following position and velocity states at cut-off time normalized to $t_o = 0$ seconds:

$$x^T(t_o) = [-3217302.678, 3527834.349, 4535013.695, -767.670, -2520.638, 5065.414]^T \quad (39)$$

where in the above, the units are in meters for position and meters/sec for velocity, respectively. The simulations of the radar case, using known BMEWS published Cobra Dane measurement covariance's for range and angle being [34]

$$\sigma_{\mathrm{range}} = 30 \text{ meters (per pulse)}; \quad (40)$$
$$\sigma_{\mathrm{angle}} = \frac{2.2}{1.6\sqrt{2 \cdot \mathrm{SNR}(t)}} \text{ degrees (per pulse)}, \quad (41)$$

[29]Notice that nothing was presumed of the estimator in deriving and evaluating Eq. 29 beyond the underlying measurement structure of Eqs. 25, 26 and the availability of all measurements **up to the current time** k. Alternative estimators that "smooth" by estimating the state x_k using measurements beyond k may violate this assumption and this CRLB but they are not real-time. The appropriate CRLB to correspond to an estimator that uses measurements beyond the current time of interest (such as in "sliding window" smoothing or in "fixed point" smoothing, or BLS) should just have the additional corresponding terms beyond the current time also included in Eqs. 31 and 32.

[30]Pre- and post-multiplying $A \geq B$ by the same matrix L yields $LAL^T \geq LBL^T$.

[31]The matrix inequality $A \geq B$ implies that $\mathrm{tr}[A] \geq \mathrm{tr}[B]$.

[32]Scalar $a \geq b \geq 0$ implies that $\sqrt{a} \geq \sqrt{b}$.

[33]This 10 MHz bandwidth Thule radar (AN/FPS-123V5), with a beamwidth of 1.8^o is located in Greenland at Latitude = 76.56^o N, Longitude = 297.70^o E. The actual range resolution is determined by beam forming to reduce sidelobes and assumptions on range accuracy of from as little as 15 meters (for the 10 MHz signal) up to more than 30 meters (for the 5 MHz signal) shouldn't significantly alter the subsequently computed results since sensitivity to the range uncertainty parameter is low as compared to the effect of the more dominant angle uncertainty.

[34]Expressed within the software in MKS units with angles in radians, respectively.

respectively [35], appear to be performing properly, as depicted in Fig. 5 for the case of a nonlinear target (corresponding to use of the system truth model used for simulating the trajectory, but linearized about the estimate within the EKF) while both situations utilized the same nonlinear measurement model. Both parameters in Eq. A.26 of [53] (with SNR varying with time) are used in Eq. A.69 of [53] with $\sigma_E \equiv \sigma_{\text{angle}}$. For position error at time t (and similarly for corresponding velocity with obvious direct replacement substitutions in the LHS of Eqs. 37 and 38), calculated as

$$\sqrt{(x_t(t) - \hat{x}(t))^2 + (y_t(t) - \hat{y}(t))^2 + (z_t(t) - \hat{z}(t))^2}, \quad (42)$$

and the corresponding ensemble sampled variance over N trials (N=250) being [36]:

$$\begin{aligned}\Sigma_N = &\left[\tfrac{1}{N}\sum_{i=1}^{N}(x_t(t) - \hat{x}_i(t))^2 + (y_t(t) - \hat{y}_i(t))^2 + (z_t(t) - \hat{z}_i(t))^2\right] \\ &- \left[\tfrac{1}{N}\sum_{i=1}^{N}\sqrt{(x_t(t) - \hat{x}_i(t))^2 + (y_t(t) - \hat{y}_i(t))^2 + (z_t(t) - \hat{z}_i(t))^2}\right]^2,\end{aligned} \quad (43)$$

were depicted for UEWR as diagrammatic plots in [22]-[24], [53] (N=1,000 in 1997 results). The subscript t appearing in both of the above equations denotes the available "truth" that is unabashedly known in simulations.

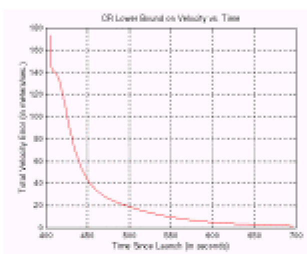

Fig. 5. CRLB on radial velocity accuracy (for Threat 1)[53]

VI. Trade-Offs between BLS and EKF

We now discuss trade-offs of Batch Least Squares (BLS) vs. Extended Kalman Filter (EKF) as these two algorithms affect radar target tracking efficacy. First, the Batch Least Squares Maximum Likelihood algorithm is familiar from being present at the core of many diverse yet familiar estimation approaches [58, Sec. 11] such as:

- the Prony method of power spectral estimation,
- some approaches to GPS Local Area Augmentation Systems (LAAS),
- within input probing for improved parameter identification [16].

The BLS that is present in all these situations has the following fundamental structure and characteristics in common, as discussed below.

[35]The radar's intrinsic range gate size dictates the effective range resolution, which is a constraint that is less restrictive than the angle acuity. The structure of the phased array radar as a measurement sensor/device having additive Gaussian measurement noise with parameters including **(a)** explicit use of the radar range uncertainty due to resolution size of the range gates and **(b)** the monopulse SNR time-record (as deduced from sum and difference channels) with its adaptive step size (as a consequence of a realistically varying PRF) as it affects the corresponding angle uncertainty.

[36]Notice that this is of the form $E[(W - E[W])^2] = E[W^2] - (E[W])^2$.

BLS use (which, as an algorithm, harkens back to Karl Frederick Gauss himself) incurs a larger computational burden than an EKF by needing a larger (and growing) CPU memory allocation to accommodate all the available sensor measurements for a particular candidate target track that are to be iteratively processed in one fell swoop over the entire time interval over which the available measurements have been accumulated and, consequently, BLS incurs more associated senescence (computational delay time that is not fixed but is also growing) than exhibited or needed by an in-place EKF (which has a delay time for computed output that is fixed and known to be on the order of n^3, where n is the state size of the EKF). Since BLS processes all the available measurements en masse and is solved iteratively over all the measurement sensor data it is provided with, the BLS algorithm may converge if the measurement data are consistent with its internal model; but if not consistent enough (as with cross target measurement mis-associations caused by crossing targets or with anomalous radar propagation characteristics due to an atmosphere that is disturbed by sunspots or by other more ominous causes), may fail to converge (a situation prudently handled by specifying a parameter LMAX as the maximum number of allowable iterations, above which BLS is treated as having NOT converged and therefore stopped; thus being prevented from running away).

The EKF immediately avails outputted estimates in a more timely fashion and tends to, more or less, follow any measurement data that it is provided with. The EKF appears to be more appropriate to use with an MTT data association algorithms [37] because it is a fixed CPU burden, which is much less than that of a BLS. On the other hand, the BLS algorithm [300] provides more accurate estimates with a higher fidelity (i.e., being more trustworthy) on-line computed covariance accompanying its estimates for the same data segment length. EKF estimation errors obtained from the on-line prediction of 1-sigma bounds were observed to be 8 times higher than the actual value (gauged against truth) for the representative scenarios that were investigated [14], [15]. The BLS on-line calculation predicts 1-σ errors of a similar magnitude but paid off by actually realizing estimation errors in the same vicinity and so possesses greater veracity in its covariance computed on-line than the standard EKF candidates discussed above (see Fig. 6).

Analyzing a variation on standard BLS use involves a slightly more complicated expression and corresponds to when BLS is called repeatedly at a known, fixed periodic rate. In this situation too, there is an upper bound worse case (conservatively arrived at to be when the BLS fails to converge) as:

$$\begin{aligned}&\text{LMAX} \cdot U \cdot \left(\sum_{i=1}^{\left[\frac{m}{r}\right]} i\right) \cdot r \\ &= \text{LMAX} \cdot U \cdot \left[\tfrac{m}{r}\right] \cdot \left(\left[\tfrac{m}{r}\right] + 1\right) \cdot \tfrac{r}{2} \ \text{ flops.}\end{aligned} \quad (44)$$

[37]Examples being *Munkres* algorithm, *generalized Hungarian* algorithm, Multiple Hypothesis Testing (MHT), *Murty's* algorithm, *Integer Programming* approach of Morefield, *Jonker-Valgenant-Castanon*, all of which either assign radar-returns-to-targets or targets-to-radar returns, respectively, like assigning resources to tasks as a solution to the *Assignment Problem* of Operations Research. Also see [85].

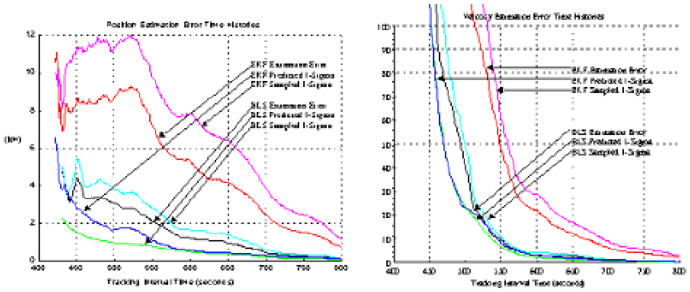

Fig. 6. Position & Velocity Track Estimation Errors vs. Time for Batch and EKF Simulations[15]

In the preceding expression, m is number of measurements and r is the period at which BLS is automatically invoked and the brackets in the upper limit of the finite summation and within the expression on the RHS (where the prior sum is simplified) denotes the smallest integer portion of the resulting division indicated to be performed within the brackets. U is the processing time required for the BLS to handle a single measurement (a per unit value).

The per measurement normalization for BLS utilized above appears to be appropriate and consistent with numerical analysis theory. The main problems being solved at the heart of each BLS iteration is the solution of a system of linear equations (the array of regression equations). Recall that this is the crux or fundamental kernel and the Householder transformation is usually used to solve it (as the algorithm of least computational complexity, which accomplishes the task at hand). Operations counts are available for a perfectly implemented sequential version of the Householder transform from [168, p. 148] ($n^2m - n^3/3$) and the associated back substitution step is O(mn), where n is the state size and m is the total number of measurements from the particular target available at that time. The CPU burden of BLS is merely linear in the number of measurements being processed.

To see how the expression of Eq. 44 was obtained, first consider the case for measurements being processed by BLS at a periodic rate where BLS is invoked after every 10 measurements (where at each invocation, all the measurements logged since the beginning for this object ID are reprocessed by BLS). For the first 40 data measurement points, where BLS was invoked after 10, after 20, after 30, and after 40, the total number of data points processed after 40 is 10 + 20 + 30 + 40 = 100. This is 100 times the measured individual per measurement CPU times discussed above. At the 47^{th} measurement, the remainder now processed is nominally no more than at 40 since the big burden of BLS processing is not invoked again until at 50 data points as:

$$\left(\sum_{i=1}^{\left[\frac{47}{10}\right]} i\right)\cdot 10 = \left[\tfrac{47}{10}\right]\cdot\left(\left[\tfrac{47}{10}\right]+1\right)\cdot\tfrac{10}{2} = 4\cdot 5\cdot 5 = 100, \tag{45}$$

where a useful formula is

$$\sum_{i=1}^{N} i = \frac{N(N+1)}{2}. \tag{46}$$

Using this result in the above CPU loading factor at time step k for a periodically invoked BLS yields a loading of:

$$\left(\sum_{i=1}^{\left[\frac{k}{c}\right]} i\right)\cdot c = \left[\frac{k}{c}\right]\cdot\left(\left[\frac{k}{c}\right]+1\right)\cdot\frac{c}{2}\ \text{flops.} \tag{47}$$

Parallel implementations should be no slower than these estimates for a Von Neumann machine and parallel multi-threaded implementations may be considerably faster; however, there are considerable parameter options and variations in a parallel mechanization such as number of processors, compatibility of algorithm architecture to parallelization, characteristics of the operating system, compiler switches, automatic distribution of algorithm over available processors or manual specification that may jeopardize expected ideal CPU timing (also see [278] for more).

This operations count goes as m (the dominant power) and again just grows linearly with m. Averaging by dividing the previous expression by m to obtain an expected per measurement normalization yields a constant based on this numerical analysis theory. A similar invocation of a Householder transformation per a measurement depicted on [168, p. 252] also obtained a constant that is a cubic in the remaining fixed variable, being n^3. All theoretical CPU flop time estimates reported here are consistent among themselves and with what was observed within the numerical computations.

In general, the more accurate 1-σ bounds from BLS help better constrain the region of space to be searched by a multi-target tracking (MTT) algorithm than would otherwise be provided by an EKF (see Fig. 6). Use of optimistic (smaller or tighter than true) 1-σ bounds usually provided by an EKF as a practical real-time sub-optimal estimator usually causes search to be more limited than prudent in Fig. 2, although supporting theoretical calculations may falsely assure success (if they expect the 1-σ available from the EKF to be trustworthy, which is usually not the case for the sub-optimal covariances provided from an EKF) while providing adequate state estimates of the target is an EKF's primary goal as a successful tracking filter (and providing covariance veracity is only secondary and is willingly sacrificed for the primary goal (notice that Zarchan et al did not even report covariances or show them in Ref. [151]). However, it is desirable to have both (and BLS does a better job at this) but EKF's are more expediently efficient.

Historically, a finite dimensional filter was sought for estimation and tracking so that it would not grow without bound as the length of the measurement data record got longer. Otherwise, the computer implementation code might overwrite itself or overwrite other critical functions (also required for mission success) that may reside on the same computer. As such, the KF processes a single measurement at a time. A trend that has been observed in investigating actual parallel implementations of other assorted algorithms over the last 20 years is that the ideal expected speed-up is seldom achieved, where a prior well-known sequential CPU time loading is anticipated to be scaled down by a factor of N or merely by a more modest squareroot of N (where N is the number of processor resources available in the particular application situation). Actual behavior also depends on the vagaries of the operating system (OS) at hand. Some OSes support parallel processing by allowing the user to stipulate where a particular module thread would reside and run. As an example, for Interactive Multiple Model (IMM) with two comparably sized filters and three available processors, having one filter per processor and the probability calculations/collations (weighted estimates and covariances) IMM output performed on its own processor, all synchronized, would be a natural fit. Other OSes no longer leave it for the user to specifically partition his own algorithm as he sees fit but instead performs this partitioning exercise for the user automatically (so the resulting fit may not be as nice as previously described). Having it done automatically is not always desirable especially if pieces of each IMM filter are distributed over all three available processors. The end result may exhibit worse CPU time than a sequentially implemented version. Performance also depends on the switch settings during compile time. See good tidings in [131] as compared to [129], [130] [38]!

In lieu of not comparing to a prior Von Neumann bound for sequential machines as a parallel implementation is sought, the alternative is to run "open loop" without any comparison of this sort. In such a case, it may be tempting to accept any performance on the parallel processors as "the best that can be done" (whether it really is or not). According to [155], the structures of "interpolating loops" more readily reap the benefits of a likely speed-up in a parallel processor implementation than "integration loops" would. Apparently an early versions of BLS used only interpolation.

VII. One Shot Success with Probability One?

What is needed in Missile Defense is an ability to achieve one shot success with probability one (instead of in terms of Mean Square Monte-Carlo averages). Researchers should pursue the work [39] of the late Frank Kozin (Brooklyn Polytechnic Univ.) [137],[160] (also see [202], [203]). Kozin sought to make strong proofs about adequate results being reaped from each and every single sample function (i.e., as probability one arguments and not just as mean square arguments). According to Kozin, earthquake resistant buildings were a consequence of Kozin's work as it related to stochastic stability. Since much of current tracker filter evaluation of utility is from Monte-Carlo runs (e.g., Farina et al use 100 trials for their accuracy and consistency evaluations in [10]; others may use 250 or 1,000), a worry is that real missile interception possesses the characteristic of being a one shot Monte-Carlo trial. There are no averages available [40] from the target tracking filter in this real-world test, just the conclusion to answer the question: did it work or didn't it? Did the tracking filter convey the correct (or adequate) coordinates of the target to the intercepting missile or not?

VIII. Use of AOT When Radar is Denied Range

Escort jammers (accompanying RV's as countermeasures) typically seek to deny target range to the observing radars but the well known and documented locations (and altitudes) of all participating EW radars can be used to an advantage to compute the baseline length between pairs of radars that can now simultaneously obtain almost synchronized views of the same targets from different perspectives and use the known observed angles to explicitly triangulate and deduce the target's range using the law of cosines (i.e., of known angle-side-angle in Fig. 7). Even better is to implicitly triangulate and calculate target range within a single angle-only tracking (AOT) filter (because it uses the history of previously observed target locations) but in order to do so, measurements from two (or more [41]) observing radars must be used simultaneously to update the filter. Historical consensus is that updating with the same measurement results from one-radar-at-a-time [42] simply will not resolve the ambiguity of many different target ranges corresponding to the same observed angle. MTT apparently still needs to be worked out for this situation and [216] and [243], [302] are good starts. AOT is even more sensitive to initial conditions than with range measurements being directly available. Target observability considerations for AOT are more challenging and are still evolving [146]-[149]. Although some analysts claim that AOT can be performed with just one sensor, the surface ship experience (Hammel and Aidala, 1981) is that with use of just one AOT sensor, the sensor platform is required to make a controlled maneuver in order to do so (while the target is assumed to be moving at a constant velocity). Alternative angles-only (a.k.a., bearings-only) filter formulations are discussed next.

Angle-only tracking (AOT) results are reported by J. R. Sklar '69; C.-B. Chang '73, '80; E. Tse, R. E. Larson, Y. Bar-Shalom, '73; R. W. Miller '78; C.-B. Chang and K.-P. Dunn

[38]Refs. [129], [130] were constrained to use existing parallel processors that had architectures that were optimized for calculating FFT's without any specific consideration of the needs for parallelizing estimation.

[39]Early on, Prof. Kozin referenced these ideas to the late Prof. J. Clifton Samuels (Purdue University, Howard University).

[40]Regarding averages, if 50% of the intercepts ended up 20-σ ahead of the target at detonation time and 50% ended up 20-σ behind it at detonation time, is that considered, on the average, right on target? Recall that in seeking to approximate a periodic square wave by its *Fourier* series representation, even if an infinite number of terms are retained in the approximation, there would be no ringing at the jump points but, despite the fact that its *Fourier* series converges in mean square to the piecewise constant periodic idealization, the well known *Gibbs phenomenon* at the jump is a spike of 30% beyond the target goal. Such are the frailties of mean and mean square convergence behavior, respectively.

[41]In planar multi-target situations, many ghost targets arise from intersecting lines of sight (as the angles of actual targets are viewed by different sensors) but can be distinguished from actual targets by using more than just two simultaneously observing sensors with their more varied perspective views. Indeed, ghosts are less likely in 3-D where the variously skewed lines-of-sight are less likely to intersect.

[42]I made this mistake 15 years ago, without warning from any mentor. However, except in simulations, it is impossible to obtain simultaneity of RV jammer target measurement reception even for bistatic situations. Simultaneous fix updating has a technical structure analogous to what arises in submarine navigation [44], [56] when 2 simultaneous navaid fixes of opportunity are taken together via different antennas, such as from both GPS and Loran-C [103].

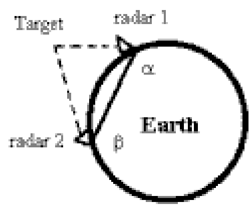

Fig. 7. Synchronous target triangulation from two (or more) radars([33])

'79; M. R. Salazar '81 [283]; C.-Y. Hsiao '88; F. D. Gorecki '91; D. V. Stallard '91; J. R. Guerci et al '94; L. G. Taff '97; and by J. P. LeCadre et al '97. An error that occurred in one of the above cited original 1973 AOT formulations, which persisted into its updated 1980 installment in IEEE AES, was discovered and corrected in [33] and the missing associated covariance (when the velocity constraints are active) was also obtained (otherwise the usual Kalman covariance should be used). The approach of [33] provides a Kalman filter formulation that removes considerable ambiguity in an RV target's motion by constraining the computed angles-only solution to lie within an acceptable range for RV velocities (similar to those mentioned in [107]). Alternative AOT approaches also exist [28]-[32] but a worry is that the batch approach of [31] may not be real-time enough. The handling of multi-targets is also aggravated in situations where angle-only information is available; however, inroads are apparently being made [279]. Other defense applications exhibiting a similar angle-only tracking geometry are: passive sonar/sonobuoy tracking using several participants, acoustic tracking of air breathing cruise missiles, and Space Based InfraRed Satellites. Once a good AOT solution is obtained, one size should fit all!

IX. Hurtles Needing Further R&D to Overcome

For the case of an **ideal linear possibly time-varying system** with additive Gaussian white process and measurement noises of known covariance intensities, with Gaussian initial condition [43], independent of the aforementioned noises and of specified mean $\bar{x}_o$ and initial covariance, P_o, and satisfying certain technical regularity conditions (of being *Completely Totally Observable* and *Controllable* [99], [201] or satisfying less restrictive, more generally met, technical conditions of being merely *Detectable* and *Stabilizable*), the following 6 properties listed below are associated with the ideal KF filter:

1) the finite dimensional n-state Kalman filter is an **optimal linear** estimator and is also the overall **optimal** estimator (according to five different statistical criteria of goodness or measures of effectiveness (MOE) listed in [18]) for tracking the state of the n-state *linear* system;
2) the estimation problem is completely solved using just the conditional mean and variance available on-line in real-time from the Kalman filter estimate and its associated Riccati equation solution, respectively. (Conditional refers to being conditioned on the sensor measurements received). Everything is Gaussian, so merely means and variances suffice;
3) there is a guarantee that the Kalman filter is stable and will converge to the true state (even if the underlying system being tracked is unstable), as has been proved using Lyapunov functions (see detailed references in [76] which explain how it was done);
4) the Kalman filter will converge exponentially asymptotically fast (this is darn quick) to the true state [115];
5) even if the initializing estimate x_o and P_o are way off (incorrect) but P_o is still *positive definite*, then the Kalman filter will still converge quickly to the **right answer** (independent of how bad the initial guess or starting values were) [115];
6) the on-line computed covariance (from the Joseph's form of the Riccati equation) is an excellent gauge or measure of how well estimation is proceeding and is even better (more accurate) in fact than statistics computed from any finite number of Monte-Carlo simulations or mission time records.

The above 6 statements are being made only for the **linear** case with known Gaussian white noise statistics and only if the filter model appropriately matches the underlying linear system model (including correctly accounting for any biases present), unlike in [116]. For nonlinear systems or non-Gaussian noises, all six [44] of the above bets are off! All are violated in general! Strategic target tracking arising in EWR typically employs nonlinear system and measurement models, as discussed in Sec. 1. Historically for EWR, EKF's have been used almost exclusively except for the $\alpha - \beta$ filters of an earlier era.

To explicitly distinguish between what to expect of a tracker for the above postulated ideal linear case and the more realistic nonlinear case encountered in practice [73], the status of approximate nonlinear filtering is discussed by paralleling the above presentation:

1) the **optimal nonlinear** filter is **infinite dimensional**, in general [45], and therefore not practical to attempt to compute (otherwise, taking possibly an infinite amount of time to do so) while a reasonable engineering approximation is to, instead, employ an Extended Kalman Filter as a **best linear** estimator (but not expected to be an optimal estimator $\hat{x}(t)$ but, hopefully, adequate for tracking the state of the nonlinear system);
2) the estimation problem is **not** completely solved using just the conditional mean and variance available on-line in real-time from the Extended Kalman filter es-

[43]Strictly speaking, even for a linear system with all the other usual conditions being satisfied, the filtering problem is infinite dimensional if the initial conditions are not Gaussian but instead belong to some other arbitrary known distribution [138].

[44]There was actually a seventh concern expressed in [58, Sec. 12] questioning the applicability of IMM for nonlinear system models and noting the apparent lack of prior precedents of IMM use with such nonlinear systems. The utility of IMM over purely KF's were recently demonstrated in [101] for certain linear systems but no nonlinear systems are treated in these comparisons of KF and IMM performance that favor IMM use. Similarly, [156] only uses linear system models for IMM even with PF that can ostensibly handle nonlinear systems and non-Gaussian noises. Also see [306].

[45]There are some limited nonlinear special cases that have finite dimensional optimal filters (as characterized by Beněs, Daum, Tam et al, Stafford [op. cit.]), where the distributions encountered within the system proper are of the *exponential family* [124, Chapts. 1-4] yet the marginal or conditional distributions may still be tractably Gaussian.

timate and its associated Riccati solution, respectively. Hopefully, such an estimate will be adequate but its intermediary variance usually is not. Unlike the situation for the linear case, where everything is completely characterized by just the estimator mean and variance, the actual optimal estimator needs all higher moments specified as well [64, Ch. 1], [65] (or, equivalently, specification of the conditional pdf or of its Fourier transform, being its characteristic function, from which moments may be generated). The on-line variance can be optimistic (smaller than actual) or pessimistic (larger than actual) and may crisscross several times over a tracking time interval between being one or the other. The primary focus is usually on the adequacy of just the state estimate as the major consideration. However, there are situations where the variance needs to be of comparable quality (see Sec. 1);

3) there is **no longer any analytically provable guarantee** that the EKF is stable and will converge to the true state. Unfortunately, EKF's sometimes diverge [254];
4) the EKF **does not** converge exponentially asymptotically fast to the true state. We are happy if it gets there fast enough to be useful;
5) when the initializing estimate x_o and P_o are way off (incorrect) but P_o is still positive definite, the EKF **may diverge** away from the right answer at an exponential rate [116]. (EKF performance can be highly dependent on how good or bad the initial guess or starting values are);
6) the on-line computed covariance (from the Joseph's form of the Riccati equation) is a **lousy** gauge or measure of how well estimation is proceeding and is **never** better (or even as accurate) as the off-line statistics computed from an adequately large finite number of Monte-Carlo simulations or mission time records. (Employing a 97% histogram-based Spherical Error Probable [SEP] from as many as 250 Monte-Carlo run evaluations is not atypical in some EWR applications.)

A desirable goal would be for researchers to eventually achieve as pleasant a situation in the above 6 categories for handling and tracking nonlinear systems as currently exists for handling the tracking of linear systems. Despite the plethora of new estimation algorithms offered and discussed in the March 2004 Special Issue of the *IEEE Proceedings* dedicated to "Estimation and Tracking" topics, it appears that not enough attention is given to the above 6 topics for the nonlinear case! Control theorists have also compiled a list of unsolved problems for esoteric abstract situations [233] yet have missed mentioning these 6 more mundane bread and butter issues that face estimation practitioners. New approaches should at least remedy one or more of the 6 problems listed above or what use are they? This same gap existed between theory and practice in 1965 and some remedies are in [241]. However, Refs. [34], [70], [104], [105], [109] [46], [121], [127] (fulfilling the promise of [128]) do appear quite lucrative and especially the milestone accomplishment of [108] for their particular imaging-based tracking solution. Ref. [108] has apparently constructively exploited every major ground breaking result in novel cutting edge random process theory that has occurred in the last 30 years (reaping structural benefits of martingale inequalities being available as associated with the greater rigor of using a Brownian motion process interpretation over merely a "Gaussian white noise" viewpoint, use of Poisson point process to capture realistic imaging aspects, use of Girsinov's transformations of measures to a computational advantage [125]) to push the envelop and reap landmark results (thus taking to fruition what Donald Snyder, Terrence McGarty, Moshe Zakai, David Sworder, Yaakov Bar-Shalom, and so many others may have had in mind as an ultimate goal for this evolving theory). This approach could eventually be as beneficial as Irving S. Reed's *streak processing* is for a similar space target application (but now no longer constrained to handling merely straight line tracks)! The new results of [172] appear to be very similar to those of Frederick E. Daum (1986, 1987). The results of [173] appear to be directly applicable to further performance analysis of a BLS batch filter (viz., CRLB). The results of [253] nicely augment the approach of [250]-[252].

For EWR, there is still room for improvement of the EKF itself including automated "process noise covariance filter tuning" [34] (recently developed and applied to GPS), [70]; possible use of iterated EKF's (of two different flavors) [35], [36]; or possible use of more terms in the Taylor series approximations of the significant nonlinearities present (constituting use of a Gaussian 2nd order filter) [3], [18], [19], [37]-[39]; and possible parallel mechanizations [40], [41] A recent evolutionary change is to use polynomial interpolations via Stirling's formula [71]or via [72] instead of evaluating any higher derivatives. All these strategies should improve the accuracy of the measurement linearization with but a slight increase in the computational burden. Manually calculating the first derivative and second derivative Jacobian [47] and Hessian matrices, respectively, was challenging 30 years ago but is quite tractable now with the advent of symbol manipulation software like Maple©, MacSyma©, or Mathematica©so [71], [72] are less enticing for this application than perhaps for others where derivatives are less readily available or

[46]While it is a laudable evolutionary service to collect and analyze the various models available for describing maneuvering targets, as done in [109], some may disagree with the remark on page 1349 that the models of Eqs. 79, 80 there are highly nonlinear. In fact, they are *bilinear* and, as such, are slightly less tractable than purely linear systems [110] (so there is no need to "pretend" in obtaining the results of Eqs. 81 and 82). Prof. Roger Brockett (Harvard) helped pioneer how to get such nice results for these almost linear systems. Alan Willsky (MIT) and David Kleinman (NPS) have stability results for these too (circa 1974) and Willsky and J.T.-H. Lo have estimation results for similar systems that have associated *Lie Algebras* that are finite dimensional (as do Beněs, Daum, Tam, Wong and Yau, Mahler [191, Chap. 5], [196], [197] also see [190], [232]). Ref. [109] missed including [111], which creatively utilizes the *Maximum Principle*. Newer approaches also exist [309].

[47]Calculating the Jacobian for a 6 state filter corresponds to forming $6^2 = 36$ derivatives that can be fairly challenging. Seeking to calculate $2^{\rm nd}$ derivative Hessians is very taxing unless symbol manipulation software is utilized.

nonexistent. Recent innovative results in contraction mapping analysis [157], [158] should be explored for likely relevance in seeking to improve theoretical underpinnings of EKF for nonlinear applications.

New exact and approximate solutions have been obtained for incorporating out-of-sequence measurements into Kalman filters [174]. This will likely be useful to compensate for transport delay incurred in cross-communications if several EWR participate to jointly track the same targets as seen from different geographical perspectives or even when augmented with the output of other types of sensors to enhance target-tracking capabilities beyond that availed from each alone. A caution is that innovative researchers sometimes use a definition of stability that differs from that classically and historically agreed upon and consistently used for decades. As a consequence, new results can validly claim to yield "stable systems" or to yield an "algorithm that converges" even though limit cycles are present (that historically would be viewed as being unstable or as an algorithm that did not converge). An earlier precedent for this situation occurring arises with use of the Min-H technique, as discussed in [44], (that is guaranteed to converge in-the-sense-of-orthogonal-search-algorithms) yet can actually vacillate and never settle down completely but, instead, can continue to hop around forever between a small finite number of equally valid options as solutions. Occasionally, the Min-H technique converges to a single unique answer and only then is its output useful as a solution to the problem at hand (as used in [44], [56]).

Rudolf Kalman used the *Hilbert Space Projection Theorem* in originally deriving the Kalman filter in 1960 (cf. [315]). The norm in L^2 is also an inner product, which is what one needs in a *Hilbert Space* (along with the space being *complete* by containing all its *limit points*). Other researchers, such as Ruth Curtin, have pursued use of *Banach space* techniques for obtaining Optimal filters for systems whose dynamics are described by Partial Differential Equations (PDE's) and whose corresponding observations constitute natural boundary conditions [266, Chap. 5]. Kalman filters for such PDE systems are also found in Andrew Sage's 1968 textbook, as identified (with applications) in [52], [58]. Randall V. Gressang and Gary B. Lamont submitted "Observers for Systems Characterized by Semi-groups," to IEEE Automatic Control in 1977 but it was rejected (not because it was wrong but because it was so far ahead of its time). Gressang and Lamont's paper posed the problem (arising for infinite dimensional systems described by PDE's) and solved it using only corresponding *Banach space* techniques (rather than use the Hilbert Space techniques that were prevalent at the time and familiar to the reviewers who failed to recognize that the Hilbert space techniques were inappropriate to use for this particular infinite dimensional situation). There is also an existing mathematical theory for handling unbounded linear operators that some believe is appropriate to use in this context when the operator at hand involves derivatives. Ref. [195] uses *Banach space* techniques when needed to handle Riccati Equations.

X. RAMIFICATIONS OF NEWER ESTIMATORS IN STOCHASTIC CONTROL? NOT YET!

We now discuss an open question that remains to be addressed for the new alternative estimation approaches. But first a short review is needed to set the context and define terms: the term LQG represents the feedback control strategy obtained by concatenating two back-to-back ideas of using a Kalman filter in conjunction with use of a Linear Quadratic (LQ) feedback control. The LQ regulator is the feedback control for driving a noise-free linear system to the zero state (termed "regulation") that minimizes or optimizes an associated convex Quadratic integral cost function (i.e., quadratic in both the state and the control), with such an endeavor being achieved over a finite time interval $[t_0, t_1]$ (for a finite planning horizon) or over an infinite interval $[t_0, \infty]$ (for an infinite planning horizon) and thus minimizing or optimizing the total energy expended in each case.

When the linear system to be regulated via a feedback control is noise corrupted, the **Separation Theorem** allows us to validly decompose the problem into the two parts mentioned above by first obtaining an estimate of the state in lieu of not having the actual noise-free state available for multiplying by the LQ feedback gain as feedback control: $u(t) = M(t)x(t)$; then we, instead, use the best available estimate of the state (being the output of the Kalman filter) in forming the corresponding LQG feedback control: $u(t) = M(t)\hat{x}(t)$. This approach is straightforward but requires solving two similar looking Matrix Riccati equations:

1) one solved forwards in time for the KF covariance used in computing the KF gain, $K(t)$, which, in turn, is used in obtaining the state estimate $\hat{x}(t)$;
2) one solved backwards in time to obtain the matrix subsequently used in computing the (possibly time-varying) LQ gain $M(t)$.

If there is negligible noise present in both the plant and in the measurement sensors, then a Luenberger Observer is utilized instead of the Kalman filter to reconstruct any unavailable states (i.e., states that are not directly accessible) for use in the feedback control.

The **Separation Theorem** (for linear, possibly time-varying, systems) supports the above described strategy by guaranteeing that the Optimal Control, which minimizes the expected value of the scalar Quadratic Convex cost function, can validly be separated into two sequentially applied parts. Unfortunately, such an easy-to-obtain solution lacks a reasonably conservative phase margin to guard against instability of this LQG control result ([88], [89]). Pure LQG solutions have a paucity (as in zero) in phase and gain margins! (As previously observed with the calculus of variations for obtaining the elusive optimum solution, it had already been observed by earlier generations of researchers that the (piece-wise continuous) time-optimal bang-bang control is also on the cusp of being unstable since a drastic instability occurs if any of the indicated "switching instants" actually implemented are even slightly offset from the ideal switching goals, and these systems can similarly go unstable even with the associated smooth LQG control strategies (cf. [119]). Loop Transfer Recovery (LTR) [90] is a further slight modification of the basic LQG methodology to force a practical solution that does have the necessary margins for safety's sake so that the resulting total feedback control solution of LQG/LTR is more robust in a changing environment (of aging hardware components resulting in slightly changing parameter values, possible presence of unmodeled high frequency dynamics unaccounted for because they did not reveal themselves as being present during the original data reduction, where the test stimulus may have been of a lesser bandwidth than needed in implementation) and, as a consequence, the LQG/LTR feedback control strategy is no longer on the cusp of going unstable, as use of LQG alone would be.

Richard Gran (retired from Grumman Aerospace and later from The MathWorks) authored [91]. W. H. Wonham (Brown University, now with Univ. of Toronto wrote in the same proceedings [92]. Many others have also participated in this quest for nonlinear separation [93]-[96].

A famous counterexample, where nonlinear separation fails, was published by H. S. Witenhausen (Bell Labs) [97]. It reveals the fallacy of attempting nonlinear separation and dashed hopes (for awhile anyway) for complete generality in the nonlinear case but engineering approximations frequently invoke this Separation procedure anyway by separating the problem of nonlinear optimal control with noise being present into two distinctly different sub-problems that are treated and solved separately, in isolation, by first performing nonlinear estimation followed by nonlinear optimal control. This two-step technique can sometimes still be useful by treating the total problem sequentially in this way although, in reality, the problem of nonlinear optimal estimation and nonlinear optimal control is inherently mixed together. Extensive simulations of the resulting algorithms are used to gauge whether it is adequate for the application at hand. More research is needed to crack this nut and reduce reliance on mere simulation (especially since no directions arise pointing to a better solution strategy as the next step to pursue if the results of the initial simulation of the separated and later combined strategy are disappointing) except [123] as a recent suggestion.

Appealing to use of the more recent so-designated H^∞ or Robust Control methodology will not likely take up the slack! (See further confirming revelations in [117, Epilogue], [118] cf. [119]) Although an H^∞ approach may perhaps be useful for process control applications, where possessing a rapid response time is not an issue because it is not sought as a goal in process control; by assuming a worse case situation for its implementation, it usually has a conservative response that is notoriously sluggish (analogous to the situation of being over-damped in 2^{nd} order linear time-invariant systems). An example supporting this assertion is that the useful and very familiar Least Mean Squares (LMS) algorithm that is known to converge [307], but sometimes slowly [308], [314], has been shown in [161] to be H^∞-optimal [48]. More to the point, the Robust Control methodology does not yet handle general time-varying linear systems, general nonlinear systems, nor systems with noises present for MIMO except in some heroic cases for a single isolated scalar system component. This is especially unsettling upon recalling that when general nonlinear systems are linearized, the result is a time-varying linear system! Recently, H^∞ control techniques have been used to detect the event of hardware component failures in systems, where the simulated failures were 1,000 times nominal; thus, use of such gigantic failure magnitudes merely conveys the impression of good performance (since results are less impressive for more reasonably sized failure magnitudes). Such demo tricks were historically warned about in [75], [141]-[143] but, evidently, still occur. Do H^∞ approaches offer solutions to any problems that could not already be solved more conventionally?

Although the late George Zames is credited in a moving (and extremely informative) tribute on pp. 590-595 in the May 1998 issue of IEEE Trans. on Automatic Control with, essentially, single-handedly bringing mathematical functional analysis to the aid of control and system theory via use of the contraction mapping principle (CMP) in [136] (see [264]), please peruse the earlier contribution by Jack M. Holtzman's (Bell Telephone Lab., Whippany, NJ) [132], which also has the use of CMP as its main theme in such systems. However, Holtzman worked everything out in detail in [132] so that his results were on a platter in such a form that they could be easily understood and conveniently applied immediately to practical system design by engineering readers faced with real applications and who may not necessarily be interested in abstract results in a technical paper whose significance is not known until several years later. Charles Desoer's and M. Vidyasagar's (U. C., Berkley) textbook came out several years earlier than Zames too and also had a functional analysis bent. A. V. Balakrishnan (UCLA) has also been an avid practitioner of functional analysis in analyzing the behavior of systems and in understanding optimal control (including numerical solution algorithms) since the early 1960's. Ref. [43, App.] even uses CMP in its convergence proof as does [55].

What about the use of *feedback linearization* or the use of neural networks for control? Answers to these questions appear in [106]. Apparently, we are still awaiting investiga-

[48]Over 12 years ago, some Japanese researchers reported getting better estimation accuracy for a somewhat uncertain system model when they used covariances, P(t), that were **not** positive definite but instead were indefinite. While such results initially appeared to be counter to prior analytic intuition, considering the important role that positive definiteness was known to play in estimation theory and LQG control and in its underlying proofs, the subsequent analysis of Refs. [162], [163] explains how this can occur when estimation is posed in a *Krein Space* rather than in a *Hilbert space*. Application results are reported in [163] but are applicable only to systems with linear time invariant (LTI) plant and measurement models but [163] provides further insight into the interrelationship with estimation and control.

tions [49] into the **downstream control impact of using the new alternative estimation approaches** of Sec. 4 in place of EKF's as the first step for actively controlling noise corrupted nonlinear systems. (See Vol. II of [64] for an alternative approach as a precedent for handling or compensating for the effect of noise on relays and on the synchronization of oscillators.) Unlike the situation for EKF's, apparently none of the new (or alternative older algorithms) "yet play a role in Stochastic Control". Evidence confirming this assertion is available by perusing the recently published Ref. [159]. Indeed, Ref. [159] elucidates a new, well-funded application area in Stochastic Control yet nary a word is mentioned about using α-β filters in the pursuit of stochastic control algorithms, nor use of Particle Filters, nor Unscented Filters (= Oxford Filters [50] = Sigma Point Filters) even though the editors of this special issue are at Oxford.

XI. Summary

We encourage investigations into new estimation approaches (such as in [182]-[185], [221]) as a way to possibly get past prior fundamental barriers that nonlinear filtering practitioners had tripped up on in the past (that we reconnoiter about in Secs. 9 and 10). However, we also desire that realistic procedures be used to evaluate the suitability of the new as well as older conventional α-β filter and BLS algorithms for specific applications, where we are only specifically concerned herein with EWR.

Covariance Intersection was shown to be wanting in Sec. 2 and questionable aspects were identified in the derivation of the Unscented or Oxford filter in Sec. 4 (footnotes). In Secs. 3 and 4, we were concerned with an apparent lack of realism relative to the EWR mission in the evaluations of [10] as it would severely affect tallies of absolute accuracy (that ignored the existence of out-of-plane errors by treating them as being essentially zero as also done in like manner in a 1998 AIAA/BMDO workshop presentation by Paul Zarchan and R. Jesionowski [313] as a possibly misleading precedent) and consequently treated by default as no longer contributing to the total error incurred in tracking. This is a serious oversight since a major part of the EWR problem, in attempting to track targets under central forces, is to identify what oscillating plane they occupy. However, since all 4 algorithms considered in [10] were evaluated under identical controlled conditions, the CPU timing and CPU loading studies of [10] for cross-comparing the algorithms were still appreciated and reported here as useful (but the state dimension of the tracker models were too low). For a higher dimensional system dynamics model of greater significance to EWR (of at least 6 states: 3 position and 3 velocity) instead of the 4 used in [10] and 5 used in what Paul Zarchan and R. Jesionowski presented [313] (in an indo-atmospheric situation where there should be at least 7 states), the CPU burden increases or blossoms nonlinearly as the model state size increases and is affected by other parameters as well (as quantified in [8] for only a particular type of Particle Filter with Bells and Whistles, as a class). We also offered warning relative to the use of α-β filters for EWR at the end of Sec. 4.

In Sec. 5, we revisited the appropriateness of an historical 30 year old approach to CRLB evaluation as still being valid for EWR. In Sec. 6, we obtained new original expressions for BLS CPU timing and loading bounds for a sequentially implemented version so that this bound may be used as a starting place gauge for comparison as more modern versions of BLS are implemented on parallel processing machines to, hopefully, reap considerable speed-ups by scaling down the CPU operations times relative to this upper bound. In Sec. 9, we reviewed the various new approaches for improving the behavior of EKF's since EKF's have historically been the workhorse in EWR and are likely to remain so for the immediate future. In Sec. 10, we noted what remains to be done before other estimation approaches fill the role that EKF's currently occupy exclusively within strategies for handling noisy control systems. Since the topics of "one-shot trials" and AOT in Secs. 7 and 8 are likely of relevance to applications beyond just EWR (as identified), we rounded out our discussion by pointing to the future with desiderata. We hope that others find our insights and comments on the field useful and relevant to their own unique estimation applications.

Appendix
Summary of Kalman Filter-Like Algorithms

As promised, here is an overview summary of the salient aspects of a Kalman filter in Fig. A.1, with structural details and consequences that can be exploited to an advantage in Figs. A.2, A.3.

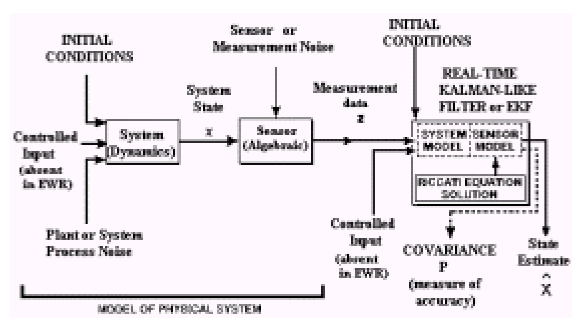

Fig. 8. Essential Aspects of using a Kalman Filter (as explained in detail in [35], [45], [51], [52], and, especially, as offered in [58] as validation cross-checks for S/W developers)

The optimal estimate is always the conditional expectation for both linear and nonlinear systems and with Gaussian or Non-Gaussian noises being present. Only for linear system

[49]We are also awaiting investigations into why Space-Time Adaptive Processing (STAP) algorithms assume enemy threat is merely stationary WGN "barrage" jamming. STAP appears to be very vulnerable to nonstationary jamming [57]. Many STAP algorithms to date (e.g., [145]) have utilized Wiener filters (which only handle time invariant situations in the frequency domain) instead of using Kalman filters (which can handle nonstationary time varying situations directly in the time domain). It is well-known that Wiener filters are a special more restrictive case of a Kalman filter [139, p.142, 242] and that MIMO Wiener filters incur the more challenging extra baggage of needing Matrix Spectral Factorization [47], [49] to take them to fruition instead of equivalently just needing to compute the more benign Matrix Riccati Equation solution. In the early 1990's in an award winning paper [144], Thomas Kailath (Stanford) and his thesis student established that most adaptive filters in current use are merely special cases of Kalman filters.

[50]Named for the affiliation of the original developers.

structures and only for exclusively additive Gaussian noises being present does the Kalman filter exactly provide such an estimate (using only a finite dimensional linear filter structure with a time-varying gain) that processes the measurements as inputs and provides this optimal estimate as output. An important aspect is that a Kalman filter can be implemented in real-time. When the underlying system and or measurement model is nonlinear or when the noises are non-Gaussian, then the structure of the filter needed to obtain an optimal estimate is, in general, nonlinear and infinite-dimensional. Since an infinite dimensional filter would be impractical to implement for real-time use, approximations are invoked and use of an Extended Kalman Filter is one such approach that yields a best linear estimate (i.e., a linear function of the measurements) that frequently provides enough accuracy to satisfy the application at hand and also involves only computations that are expediently real-time. A nonlinear estimator could ostensibly be more accurate (but would likely be impractical to pursue further because of a likely exorbitant computational burden that would preclude being real-time). Novel EKF generalizations for non-Gaussian noises are [204]-[206].

From Fig. A.1, it is seen that when the original system is linear (possibly time-varying) and all noises are WGN, then the linear system structure of the available measurements feeding into a Kalman filter (with time-varying gains) for further processing preserves Gaussianess throughout since linearity is preserved throughout. Gaussian processes are completely characterized by just their mean and variance. The Kalman filter is just an efficient computational algorithm for generating both the conditional mean and its associated accompanying covariance in real-time. Use of Extended Kalman filters (EKF) is one way to attempt an approximation for handling nonlinear and or non-Gaussian applications in the same way. Sometimes such EKF use is adequate. Several engineering refinements to a basic EKF are described in this paper as relevant to EWR use.

Both refs. [175], [176] present insightful cutting edge results from probability and statistics, already laid out within a Kalman filtering context and tailored to estimation applications, that pertain specifically to extending KF applicability to situations involving noises from exponential families and in seeking out sufficient statistics that, by capturing the available information in the most compact way, minimizes the complexity incurred in algorithm implementation. See [180]. Jerry Mendel and Max Nikias trail blazed with many published papers and a book (along with a short course in the 1990's through their company: Circuits and Systems Inc.) on α-stable noises and "stack filters" (and on sorting out bispectra and trispectra approaches for multidimensional/multichannel real and/or complex random processes in engineering systems) yet their breakthrough work done decades earlier is not referenced in recent papers on these same topics [214], [215]. Other rigorous approaches that appear to be very useful arise in [235]-[240].

While Extended Kalman Filters are model-based (as is the Kalman filter for the purely linear case), both are generally applied to state variable representations of the system, which for radar target tracking usually models the target dynamics (and any maneuvers anticipated for the particular application). A generalization of the standard state variable representation is to model the system in terms of so-designated *descriptor* systems (DS), where simplifications frequently accrue that reduce the computational burden associated with implementation of the appropriate estimator corresponding to the descriptor structure of the dynamics model [198]-[200], [226], [246]-[248], [276], [280]. When these types of descriptor model representations are utilized as a more natural fit of the physical system to its software implementation with estimation algorithms, the block-by-block connection diagrams such as those used by The MathWorks' Simulink©(or IBM's CSMP©), as a throw-back to the technology of 30 years ago are no longer necessary, as was originally clearly explained in [209], [210]. Such a descriptor systems approach avoids or side steps the need for special high CPU overhead algorithms for integrating "stiff differential equations" (typically done for block-by-block representations via use of Gear's implicit integration routines [211]), as touted for MatLab©/Simulink©in [212]. Descriptor system representations decompose the short circuit fast loop or short time constant into an algebraic equation devoid of dynamics along with a lower dimensional residual dynamics representation. Both of these operations reduce the complexity in adequately representing such otherwise "stiff" systems and, moreover, frequently obviate the need for special Gear-type implicit integration algorithms altogether in these particular situations since simpler Runge-Kutta predictor-corrector algorithms frequently suffice. However, in systems that have a residual wide range of effective time constants (even after algebraic loops are removed) or fast inner and slow outer control loops present (as with fighter aircraft guidance laws) or because of the presence of multiple sampling rates, Gear-type integration may sometimes still be needed, but now less frequently and more parsimoniously invoked.

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Extended **K**alman **F**ilter software implementations are applicable to systems whose state variable representation is a **Nonlinear System** (which means not necessarily linear and can include linear systems as a special case).

Process or Plant noise (usually Gaussian White Noise)

Deterministic Control input (exogenous variable)

System Dynamics:

$$\dot{\mathbf{x}}(t) = \mathbf{a}(\mathbf{x}(t), t, \mathbf{w}(t), \mathbf{u}(t)),$$

with initial condition $\mathbf{x}(0) = \mathbf{x}_o$

ODE of this form is **too general** to be *easily tractable*

Sensor Measurement: $\mathbf{z}(t) = \mathbf{c}(\mathbf{x}(t), t, \mathbf{v}(t))$,

Initial condition is a random variable (usually Gaussian)

where t is the scalar independent variable of time and $\dot{\mathbf{x}} \triangleq \frac{d\mathbf{x}}{dt}$

(with known mean: $\bar{\mathbf{x}}_o$ and variance: $\mathbf{P_o}$)

Measurement noise (usually Gaussian White Noise)

Slightly less general ODE **is tractable** as: **Nonlinear Non-autonomous (**time t **appears** explicitly in **System Dynamics)**

System Dynamics:

$$\dot{\mathbf{x}}(t) = \mathbf{a}(\mathbf{x}(t), t) + \mathbf{f}(\mathbf{x}(t), t)\mathbf{w}(t) + \mathbf{b}(\mathbf{x}(t), t)\mathbf{u}(t),$$

with initial condition $\mathbf{x}(0) = \mathbf{x}_o$ (as above)

Sensor Measurement: $\mathbf{z}(t) = \mathbf{c}(\mathbf{x}(t), t) + \mathbf{g}(\mathbf{x}(t), t)\mathbf{v}(t)$

Second order statistics of both Process and Measurement **noises** above must be completely known (as either the mean and correlation function matrix or mean and power spectrum matrix) and be statistically independent of $\mathbf{x_o}$.

Slightly less general ODE **is tractable** as: **Nonlinear Autonomous (**explicit time t **is absent** in **System Dynamics)**

a, **f**, **b** are each assumed to satisfy a local *Lipschitz* condition for these less general ODE's to have solutions that *exist* and are *unique*. **c** and **g** are assumed *continuous* in time t. (Specifying that **a**, **b**, **c** be merely *continuous* suffices to guarantee ODE solutions *exist* but may not be *unique*.)

System Dynamics:

$$\dot{\mathbf{x}}(t) = \mathbf{a}(\mathbf{x}(t)) + \mathbf{f}(\mathbf{x}(t))\mathbf{w}(t) + \mathbf{b}(\mathbf{x}(t))\mathbf{u}(t),$$

with initial condition $\mathbf{x}(0) = \mathbf{x}_o$ (as above)

Sensor Measurement: $\mathbf{z}(t) = \mathbf{c}(\mathbf{x}(t)) + \mathbf{g}(\mathbf{x}(t))\mathbf{v}(t)$

and an **Extended Kalman Filter** (EKF) may be used to estimate the state **x**(t) (and similarly for the **non-autonomous** situation on the left).

After linearization, this has **Time-Varying Parameters** (TVP)

System Dynamics:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{F}(t)\mathbf{w}(t) + \mathbf{B}(t)\mathbf{u}(t),$$

with initial condition $\mathbf{x}(0) = \mathbf{x}_o$

Sensor Measurement:

$\mathbf{z}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{G}(t)\mathbf{v}(t)$, and transition matrix is obtained by integrating $\frac{\partial}{\partial t}\mathbf{\Phi}(t,s) = \mathbf{A}(t)\mathbf{\Phi}(t,s)$, for $\mathbf{\Phi}(s,s) = \mathbf{I}_{n \times n}$ for any s

After linearization, this is **Linear Time Invariant** (LTI)

System Dynamics:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{F}\mathbf{w}(t) + \mathbf{B}\mathbf{u}(t),$$

with initial condition $\mathbf{x}(0) = \mathbf{x}_o$

Sensor Measurement:

$\mathbf{z}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{G}\mathbf{v}(t)$, and transition matrix simplifies to be just the matrix exponential: $\Phi(t,s) = \mathbf{e}^{\mathbf{A}(t-s)}$ for any s, t

(If original system is of this form and technical regularity conditions of detectability/stabilizability are satisfied, then a mere **Kalman Filter** suffices.)

(Continued below)

Figure 9: *Structural Aspects of a System's Model can be exploited to an Advantage—Part I*

System Dynamics:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{F}(t)\mathbf{w}(t) + \mathbf{B}(t)\mathbf{u}(t),$$

with initial condition $\mathbf{x}(0) = \mathbf{x}_o$

Sensor Measurement:

$\mathbf{z}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{G}(t)\mathbf{v}(t)$, and transition matrix is obtained by integrating $\frac{\partial}{\partial t}\mathbf{\Phi}(t,s) = \mathbf{A}(t)\mathbf{\Phi}(t,s)$, for $\mathbf{\Phi}(s,s) = \mathbf{I}_{n\times n}$ for any s

(If original system is already of this form and controllability/ observability conditions hold, then a mere **Kalman Filter** suffices.)

After linearization, this is **Linear Time Invariant** (LTI)

System Dynamics:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{F}\mathbf{w}(t) + \mathbf{B}\mathbf{u}(t),$$

with initial condition $\mathbf{x}(0)=\mathbf{x}_o$

Sensor Measurement:

$\mathbf{z}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{G}\mathbf{v}(t)$, and transition matrix simplifies to be just the matrix exponential: $\Phi(t,s) = \mathbf{e}^{\mathbf{A}(t-s)}\ \forall s,t$

Solution representation (for asynchronous time steps):

$$\mathbf{x}(t) = \Phi(t,s)\mathbf{x}(s) + \int_s^t \Phi(t,\tau)[\mathbf{F}(\tau)\mathbf{w}(\tau) + \mathbf{B}(\tau)\mathbf{u}(\tau)]d\tau$$

then let $t = t_{k+1}$ and $s = t_k$, thus we have

$$\mathbf{x}(t_{k+1}) = [\mathbf{\Phi}(t_{k+1},t_k)]\mathbf{x}(t_k) + \mathbf{w}'(t_k) + \bar{\mathbf{B}}(t_{k+1},t_k)\mathbf{u}(t_k),$$

(where $\mathbf{u}$ is assumed to be constant over the interval from t_k to t_{k+1})

Now the above is of the form of a time-varying, discrete-time linear system and digital electronically sampled measurements are usually only available at discrete instants of time as:

$$\mathbf{z}(t_k) = \mathbf{C}(t_k)\mathbf{x}(t_k) + \mathbf{G}(t_k)\mathbf{v}(t_k)$$

Assumption that control $\mathbf{u}$ is constant over these brief time intervals can be *enforced* using a zero order hold **(ZOH),** otherwise, may consider time interval to be taken to be **short enough** for this assumption to be true.

Solution representation (for most general asynchronous time steps):

$$\mathbf{x}(t) = \mathbf{e}^{\mathbf{A}(t-s)}\mathbf{x}(s) + \int_s^t \mathbf{e}^{\mathbf{A}(t-\tau)}[\mathbf{F}w(\tau) + \mathbf{B}u(\tau)]d\tau$$

then let $t = t_k$ and $s = t_{k+1}$, thus we have

$$\mathbf{x}(t_{k+1}) = \left[\mathbf{e}^{\mathbf{A}(t_{k+1}-t_k)}\right]\mathbf{x}(t_k) + \mathbf{w}'(t_k) + \left[\int_{t_k}^{t_{k+1}} \mathbf{e}^{\mathbf{A}(t_{k+1}-\tau)}\mathbf{B}d\tau\right]\mathbf{u}(t_k) \leftarrow \text{(assumed to be constant over the interval)}$$

which is of the form of a time-varying, discrete-time linear system:

$$\mathbf{x}(t_{k+1}) = \bar{\mathbf{A}}(t_{k+1} - t_k)\mathbf{x}(k) + \mathbf{w}'(t_k) + \bar{\mathbf{B}}(t_{k+1},t_k)\,\mathbf{u}(t_k),$$

and digital electronically sampled measurements are usually only available at discrete instants of time as:

$$\mathbf{z}(t_k) = \mathbf{C}\mathbf{x}(t_k) + \mathbf{G}\mathbf{v}(t_k);\text{ and for } \mathbf{w}' \text{ the effective } Q'(t_{k+1},t_k) \triangleq \int_{t_k}^{t_{k+1}} \mathbf{e}^{\mathbf{A}(t_{k+1}-\tau)}\mathbf{F}Q\mathbf{F}^{\mathrm{T}}\mathbf{e}^{\mathbf{A}^{\mathrm{T}}(t_{k+1}-\tau)}d\tau$$

Solution representation when there is a <u>synchronous time-step</u> Δ throughout: Accrues **No** additional structurally beneficial simplifications other than those already indicated above.

Solution representation when there is a <u>synchronous time-step</u> Δthroughout:

Let $t = (k+1)\cdot\Delta$ and $s = k\cdot\Delta$, thus we have $\mathbf{x}(t_{k+1}) = \left[\mathbf{e}^{\mathbf{A}\cdot(t_{k+1}-t_k)}\right]\mathbf{x}(t_k) + \mathbf{w}'(t_k) + \bar{\mathbf{B}}(t_{k+1} - t_k)\mathbf{u}(t_k)$,

which is of the form of a time invariant, discrete-time linear system:

$$\mathbf{x}(k+1) = \bar{\mathbf{A}}(\Delta)\mathbf{x}(k) + \mathbf{w}'(t_k) + \bar{\mathbf{B}}(\Delta)\mathbf{u}(t_k) \Rightarrow \mathbf{x}(k+1) = \bar{\mathbf{A}}\mathbf{x}(k) + \mathbf{w}'(k) + \bar{\mathbf{B}}\mathbf{u}(k),$$

and digital electronically sampled measurements are usually only available at discrete instants of time as:

$$\mathbf{z}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{G}\mathbf{v}(k);\text{ and for } \mathbf{w}' \text{ the effective } Q' \triangleq \int_0^{\Delta} \mathbf{e}^{\mathbf{A}\tau}\mathbf{F}Q\mathbf{F}^{\mathrm{T}}\mathbf{e}^{\mathbf{A}^{\mathrm{T}}\tau}d\tau \text{ (a constant matrix)}$$

Figure 10: *Structural Aspects of a System's Model can be exploited to an Advantage—Part II*

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PLACE PHOTO HERE

Thomas H. Kerr III received the BSEE (magna cum laude) from Howard Univ. in 1967 and MSEE and Ph.D. degrees in electrical engineering from the Univ. of Iowa, Iowa City in 1969 and 1971, respectively, both in control and estimation.

Dr. Kerr's experience for the past 35+ years as an R&D algorithm engineer and software developer has encompassed various Kalman filter theoretical evolutionary developments for DoD applications. As a Controls Engineer at General Electric's Corporate Research & Development Center (Schenectady, NY; 1971-'73), he worked on steam car and other simulations in, analysis for, and improvements to G.E.'s Automated Dynamic Analyzer (ADA). At The Analytic Sciences Corporation (Reading, MA; 1973-'79), he developed real-time failure detection algorithms for Poseidon and Trident SSBN submarine SINS/ESGM navigation and evaluated/improved algorithms and strategies for SSBN navaid fix-taking. At Intermetrics, Inc. (Cambridge, MA; 1979-'86), he was involved in applying evolving decentralized Kalman filters for Navy JTIDS RelNav refinement and for Air Force ICNIA decentralized filter development, along with performing IV&V for sonobuoy and DT&E(OR) for GPS in the SSN-701 attack sub. At Lincoln Laboratory of MIT (Lexington, MA; 2986-'92), he performed radar target tracking of reentry vehicles (RV's), developed credible RV decoy wake signatures, and performed GPS navigation analysis for airborne terrain mapping in support of a Neural Network-based ATR application (by others). He is currently CEO/Principal Engineer at TeK Associates, an engineering consulting, R&D, and s/w development company he founded in 1992. He taught *Optimal Control* in the Graduate ECE Department of Northeastern Univ. in the evenings for five years (1990-'95).

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He received the M. Barry Carlton Award for Outstanding Paper to appear in *IEEE Trans. on Aerospace and Electronic Systems* for 1987, has been chairman of the local Boston IEEE Control Systems Section twice (1990-92; 2002-04), is a senior member of both the IEEE and the AIAA, and is a member of the Institute of Navigation (ION), ISA, and MSDN (level 2) as well as a life Member of the National Defense Industrial Association. Academic affiliations: ΤΒΠ, ΠΜΕ, ΣΠΣ, ΕΚΝ, and ΣΞ. He is listed in *Who's Who* in the East ('92), Technology ('93), the World ('98), the USA ('03), Finance and Business ('05), America ('06), Science & Engineering ('06). He has done 12 marathons and bikes regularly with CRW (since '77).