New lamps for Old: A Shell Game for Generalized Likelihood Ratio use in Radar? Or this isn't you father's GLR!*

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ABSTRACT

We alert the reader here to a discrepancy between what has recently been referred to as the Generalized Likelihood Ratio (GLR) approach to radar target detection and what has historically been used as the GLR approach to this same problem in detection theory. Despite these identified discrepancies, the recent version is tractable and has desirable properties and consequently exhibits behavior that is very encouraging. After making the necessary clarifications, we summarize the status of the new pseudo-GLR and contrast it to what was available from the older (evidently antiquated) literature on GLR formulation, which, however, did serve as an historical precedent.

Keywords: Detection Theory From First Principles, Statistical Basis of Generalized Likelihood Ratio Hypotheses Tests, Mixed Hypotheses GLR, Radar Target Detection and Tracking, Nonlinear Filtering, Growing Banks-of-Kalman Filters (not IMM), Central Limit Theorem (CLT) Update

1. INTRODUCTION

Harry L. Van Trees' definition of GLR (on p. 92 and p. 366 in [1]) and that of Fred C. Schweppe [2] involves taking the usual likelihood ratio and substituting back into it by replacing ALL unknown parameters with their maximum likelihood estimates. Unlike the ordinary likelihood ratio, which is a uniformly most powerful (UMP) test and is a sufficient statistic for the decision (by being factorizable according to the Rao-Blackwell theorem [3, p. 155]), GLR's have no such optimality claim but can still be a good idea to use. The REAL PROBLEM is to look at ALL the sampled measurements of the received random process over the time interval $t_o < t_i < t_f$ and then decide whether it is representative of just the stationary white measurement noise random process being present or is it representative of a deterministic signal also being present and embedded in that noise. The deterministic signal has a known structure but perhaps unknown scale (magnitude or intensity) that can be included as a parameter to estimate. The known signal can have different candidate start times and the classical GLR includes signal start time as one of those parameters to maximize over in the GLR to obtain its maximum likelihood estimate.

Despite my allegiance to the historical GLR definition of the 1960's and 1970's, I acknowledge that the new GLR version first appearing in the mid 1980's and, as recently embraced by many researchers [11]–[17], is interesting and fun too. The recent papers on the new GLR exhibit great mathematics and statistics and contain clear discussions (except that they fail to make the connection with the historical GLR formulation for the radar problem, as in C. W. Helstrom's 1960's book [82]). I will summarize both the old and these new results below.

2. HISTORY OF GENERALIZED LIKELIHOOD RATIO (GLR) USE: FIRST IN RADAR THEN IN NAVIGATION FAILURE DETECTION APPLICATIONS

As mentioned in the Introductory section, the REAL PROBLEM is to look at ALL the sampled measurements of the received random process over the time interval $t_o < t_i < t_f$ and decide whether it is representative of just the stationary white measurement noise random process as:

H_o: received measurements being $z(t_i) = n(t_i)$ for $t_o < t_i < t_f$;

or is there a deterministic signal also present and embedded in that noise as:

H_1: received measurements being $z(t_i) = s(t_i) + n(t_i)$ for $t_o < t_i < t_f$.

Research funded by TeK IRD Contract No. 2001-103.

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Signal and Data Processing of Small Targets 2001, Oliver E. Drummond, Editor, Proceedings of SPIE Vol. 4473 (2001) © 2001 SPIE · 0277-786X/01/$15.00

Again, the deterministic signal has a known structure but perhaps unknown scale (magnitude or intensity) that can be included as a parameter to estimate. The known signal can have different candidate start times and the classical GLR includes signal start time as one of those parameters to maximize over in the GLR to obtain its maximum likelihood estimate. So H_1 is really a compound or mixed hypothesis rather than merely a simplistic binary hypothesis test. (This formulation is also discussed in Jack K. Wolf's Princeton '59 Ph.D. thesis [4] and by J. B. Thomas and E. Wong [5], by W. B. Davenport and W. L. Root's '58 textbook [6], and in Root [7], and in I. Selin [8], [9]. Robert McAulay (Lincoln Laboratory of MIT) and E. Denlinger received the Barry Carlton Award from IEEE AES for this original GLR work (using the above definition of GLR) for [10]. **Problem ONE:** the idealized GLR mechanization for this GLR formulation corresponds to a growing bank-of-Kalman-filters, with a new one spawned for each new candidate signal start time [34]. (Of course, only approximate implementations can be realized and these approximations can be off by orders of magnitude from the ideal GLR in the calculation of the test statistic's true value [29, App. A, pp.973-4]). **Problem TWO:** the specification of the decision threshold wasn't tractable (but standard Chi-squared approximation could still be invoked but problem ONE still dominates the situation [34]).

Based on my prior experience [34] and being aware of Ed Kelly's GLR work at Lincoln Laboratory [11], I saw in 1989 that his formulation was not the original GLR formulation for time segments of random processes (which is how the clean problem was originally phrased or casted). However, Kelly's test statistic does appear to be a more tractable approximation to the Original GLR formulation but ignores consideration of the time interval (yet estimated covariance is obtained from samples over a time interval but not necessarily the exact same time interval since its specification is left somewhat open by not being explicitly delimited beforehand).

Alan Willsky (MIT) and Hal Jones (TASC) applied this GLR to failure detection in navigation systems for the Air Force in 1975 (IEEE AC) [18], [19]. K.P. Dunn and C. Bing Chang (L.L.) applied this to tracking maneuvering targets [20]. Jack Liu (TASC) also applied GLR to navigation systems [21]. Also see [22]. All of the above formulations are for linear (possibly time-varying systems) only.

I looked into all of these Air Force solutions for the Navy in the 1970's. I ultimately used a different, more tractable, more robust solution based on confidence regions for the same type of Navy problem (for failure detection in navigation systems). Here is what I used and the application for which it was performed [23]–[28], [30]–[34].

I also found evidence of tampering in prior presentations of GLR results. Decision thresholds were superimposed after the fact (with times of disturbance occurrence known by the researcher beforehand)-NOT a double blind study, as would now be required to pass.

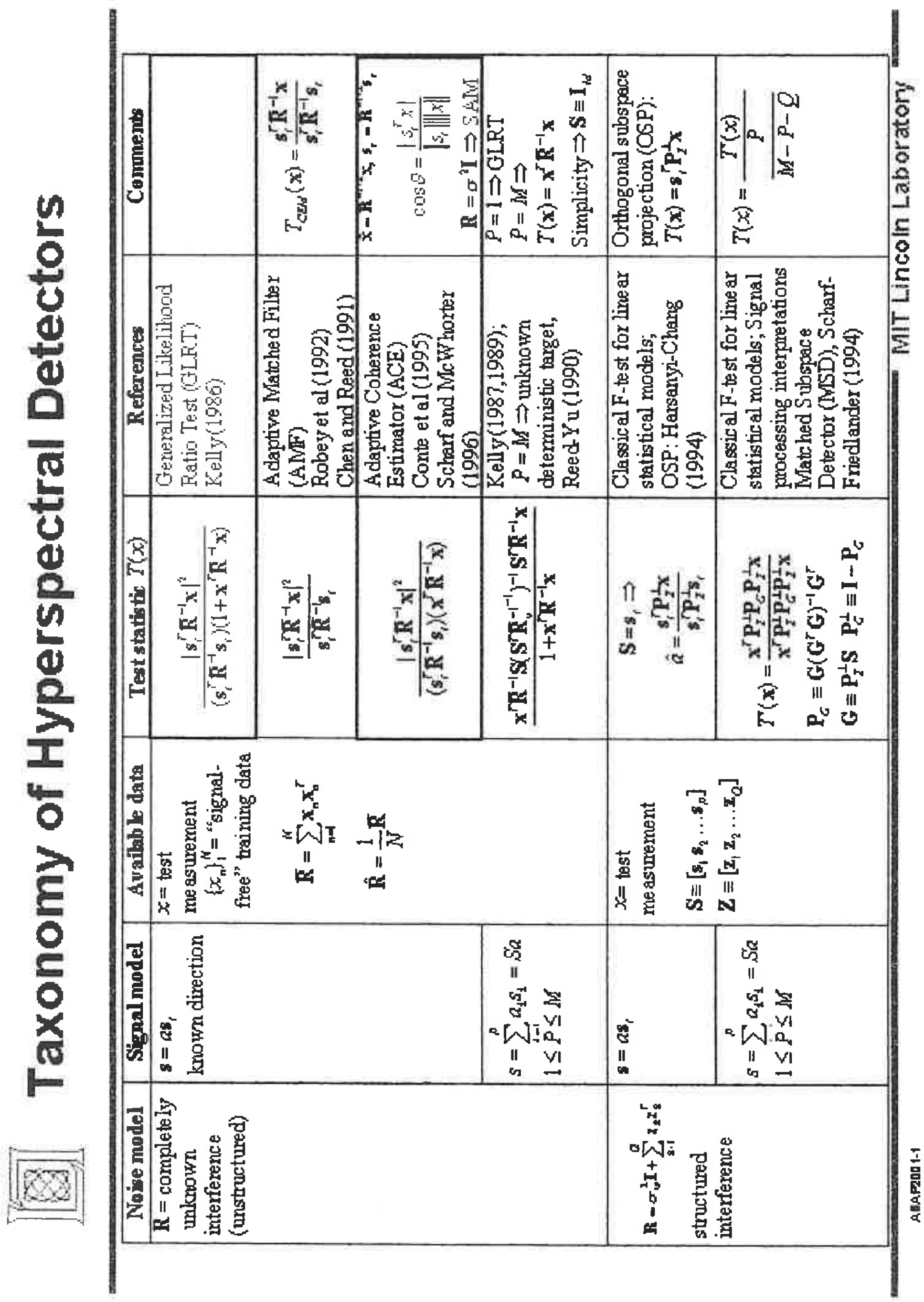
Detailed perspectives on the above may be found in [34]. A summary of the above perspectives may be found within [53], [54].

As mentioned on p. 366 of Vol. I of Van Trees [1], Carl W. Helstrom's book [82] discusses the application of the above described GLR to the radar problem of detecting signals of UNKNOWN arrival time.

Recent (since 1997) speech recognition work by Prof. lass in AI Lab at MIT uses GLR without consideration of any decision threshold at all. They just look for relative spikes up that may occur in time as their clue of when new phenome events of significance occur in speaker independent speech. Fig. 1 was presented by Lincoln laboratory of MIT at the ASAP Symposium this past spring.

Some analyst, like Dr. Anthony Filip (Lincoln Labratory of MIT), say that radar is willing to tolerate the current GLR approximation now in vogue in order to gain the tractability but gives up on the explicit exactitude of signal start time. The signal start time is critical in failure detection (and I suspect in target maneuver detection too) because that is when other systems are switched in and the system is reconfigured with redundant or warm standby components or by analytical redundancy (i.e., use of combination of other remaining existing unfailed sensors-perhaps with further processing-to fill the void left by the failed sensor) to remedy the situation, as elaborated upon in more detail in [54].

One Russian researcher (now in the U.S.), I. V. Nikiforov, continues to use the rigorous formulation and recently obtained tractable finite implementation results for even nonlinear detection situations [83]. Nikiforov has a prior book in English on this same subject [84]. His notation and concepts can be challenging for engineers. He's a martingale, measure theory person (as needed in this subject matter) to be entirely rigorous (as known by probability theorists since Kolmogorov's 1933 work), which he is. Both these publications extend the applicability of useful analytic results from [85] for random processes..



Noise model	Signal model	Available data	Test statistic $T(x)$	References	Comments
$\mathbf{R}$ = completely unknown interference (unstructured)	$s = a s_t$ known direction	x = test measurement $\{x_n\}_1^N$ = "signal-free" training data $\mathbf{R} = \sum_{n=1}^{N} \mathbf{x}_n \mathbf{x}_n^T$ $\hat{\mathbf{R}} = \frac{1}{N}\mathbf{R}$	$\frac{\lvert s_t^T \mathbf{R}^{-1}\mathbf{x}\rvert^2}{(s_t^T \mathbf{R}^{-1} s_t)(1+\mathbf{x}^T \mathbf{R}^{-1}\mathbf{x})}$	Generalized Likelihood Ratio Test (GLRT) Kelly(1986)	
			$\frac{\lvert \mathbf{s}_t^T \mathbf{R}^{-1}\mathbf{x}\rvert^2}{\mathbf{s}_t^T \mathbf{R}^{-1}\mathbf{s}_t}$	Adaptive Matched Filter (AMF) Robey et al (1992) Chen and Reed (1991)	$T_{CEM}(\mathbf{x}) = \frac{\mathbf{s}_t^T \mathbf{R}^{-1}\mathbf{x}}{\mathbf{s}_t^T \mathbf{R}^{-1}\mathbf{s}_t}$
			$\frac{\lvert \mathbf{s}_t^T \mathbf{R}^{-1}\mathbf{x}\rvert^2}{(\mathbf{s}_t^T \mathbf{R}^{-1}\mathbf{s}_t)(\mathbf{x}^T \mathbf{R}^{-1}\mathbf{x})}$	Adaptive Coherence Estimator (ACE) Conte et al (1995) Scharf and McWhorter (1996)	$\tilde{\mathbf{x}} = \mathbf{R}^{-1/2}\mathbf{x},\ \tilde{\mathbf{s}}_t = \mathbf{R}^{-1/2}\mathbf{s}_t$ $\cos\theta = \frac{\lvert \tilde{s}_t^T \tilde{x}\rvert}{\lVert \tilde{s}_t\rVert \lVert \tilde{x}\rVert}$ $\mathbf{R} = \sigma^2\mathbf{I} \Rightarrow$ SAM
	$s = \sum_{i=1}^{P} a_i s_i = Sa$ $1 \le P \le M$		$\frac{\mathbf{x}^T\mathbf{R}^{-1}\mathbf{S}(\mathbf{S}^T\mathbf{R}_v^{-1})^{-1}\mathbf{S}^T\mathbf{R}^{-1}\mathbf{x}}{1+\mathbf{x}^T\mathbf{R}^{-1}\mathbf{x}}$	Kelly(1987,1989); $P = M \Rightarrow$ unknown deterministic target, Reed-Yu (1990)	$P = 1 \Rightarrow$ GLRT $P = M \Rightarrow$ $T(\mathbf{x}) = \mathbf{x}^T\mathbf{R}^{-1}\mathbf{x}$ Simplicity $\Rightarrow \mathbf{S} \equiv \mathbf{I}_M$
$\mathbf{R} = \sigma_v^2\mathbf{I} + \sum_{k=1}^{Q} \mathbf{z}_k \mathbf{z}_k^T$ structured interference	$s = a s_t$	x = test measurement $\mathbf{S} \equiv [\mathbf{s}_1\ \mathbf{s}_2 \ldots \mathbf{s}_p]$ $\mathbf{Z} \equiv [\mathbf{z}_1\ \mathbf{z}_2 \ldots \mathbf{z}_Q]$	$\mathbf{S} = \mathbf{s}_t \Rightarrow$ $\hat{a} = \frac{\mathbf{s}_t^T \mathbf{P}_Z^{\perp}\mathbf{x}}{\mathbf{s}_t^T \mathbf{P}_Z^{\perp}\mathbf{s}_t}$	Classical F-test for linear statistical models; OSP: Harsanyi-Chang (1994)	Orthogonal subspace projection (OSP): $T(\mathbf{x}) = \mathbf{s}_t^T \mathbf{P}_Z^{\perp}\mathbf{x}$
	$s = \sum_{i=1}^{P} a_i s_i = Sa$ $1 \le P \le M$		$T(\mathbf{x}) = \frac{\mathbf{x}^T\mathbf{P}_Z^{\perp}\mathbf{P}_G\mathbf{P}_Z^{\perp}\mathbf{x}}{\mathbf{x}^T\mathbf{P}_Z^{\perp}\mathbf{P}_G^{\perp}\mathbf{P}_Z^{\perp}\mathbf{x}}$ $\mathbf{P}_G \equiv \mathbf{G}(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T$ $\mathbf{G} \equiv \mathbf{P}_Z^{\perp}\mathbf{S}\ \ \mathbf{P}_G^{\perp} \equiv \mathbf{I} - \mathbf{P}_G$	Classical F-test for linear statistical models; Signal processing interpretations Matched Subspace Detector (MSD), Scharf-Friedlander (1994)	$T(x) = \frac{T(x) / P}{M-P-Q}$

Figure 1. Lincoln laboratory Overview of GLR, as it arises in hyperspectral processing. (Compare with similar results and tables in [44])

The "new GLR" is but an approximation to the original because they choose to ignore the explicit time interval. They don't acknowledge their "GLR" to be an approximation to the true historically enunciated GLR (of Jack Wolf, of J. B. Thomas and Eugene Wong, of Carl W. Helstrom, of Harry L. Van Trees, of Fred C. Schweppe, of Robert McAuley, of Alan S. Willsky). They don't acknowledge the historical true GLR at all. However, this "new GLR" has many nice analytic properties that have been established by top-notch analysts (a veritable who's who of the analytically gifted in the detection area now such as: Ed. J. Kelly, Irving S. Reed, Donald Tufts, Jaime R. Roman, Muralidhar Rangaswamy) have confirmed the utility of the "new GLR" formulation and its variants with both simulated data and with limited real data [11]–[17]. [3] has an example.

3. A REASON FOR OPTIMISM REGARDING THE NEW GLR'S PRACTICALITY FOR RADAR (AND PERHAPS OTHER) APPLICATIONS

The very pertinent comments of this section are by Dr. Jaime Roman: "With respect to the technical comments that you sent, I agree with you on the GLR issue. For some reason the connection (similarities/differences) has never been made with the much older work, even though Kelly and his coworkers must have been aware of it. In the work I have been involved with for the Air Force (the team of co-authors in the paper and a few others), I have been using detection rules other than the GLRT, but of similar form. They do work pretty well, as evidenced in the work the team has presented and that you reference. I think of our approach as a model-based approach, and that is the main unique aspect in the radar application. It turns out, however, that a more generalized model-based methodology is relevant to various other applications (as well).

Another point that you raise has to do with the initial time of an event. The radar application that has been pursued by the crowd you refer to is (one of) long-range airborne radar surveillance. In that context, the fast-rate processing is carried out in "coherent processing intervals (CPIs)" of short duration in relation to the radar footprint, typical target velocities, radar range resolution, target size, and other parameters such that it is usually acceptable to miss initiation of the target track by one CPI (assuming that the target's return is strong enough such that its return will integrate to a value above the detection threshold once it is present over a full CPI). Target tracking is a slow-rate processing task that is carried out using the fast-rate detection outputs."

4. CONCLUSIONS

To avoid confusion with the earlier rigorous statistics-based definition of Generalized Likelihood Ratio (GLR), the critical reader should note that since the so-called "new GLR" does not abide by the fundamental definition of a GLR, which requires use of only maximum likelihood estimates for all the unknown parameters occuring in this problem, it has violated the technical definition of what a GLR is and therefore should no longer be allowed to use this same name. Statisticians would be "up in arms" about this laughable notational violation and flagrant disregard of their careful definitions and again (as before) suspect that engineers don't know what they are talking about by abusing their definitions like this. (Statisticians now after 30 years have been won over to the engineering use of Extended Kalman Filters and their important role in engineering problems and in parameter estimation for random processes.) Upon now being sensitized to the conflict with careful statistical nomenclature, engineers should realize the situation and be willing to give this new, well-defined test statistic a new name that won't be confused by statisticians with their GLR, since they named it GLR first to denote a well-defined unambiguous concept. We are corrupting the science by being sloppy with our naming conventions even thought our underlying concepts are correct. That's how we create wars with statisticians for no good reason. Do we crack the egg on the small end or on the large end? Or am I half cracked and half baked? I'll get off the soap box now. "Repent the end is near!"

ACKNOWLEDGMENTS

My insights into the jamming vulnerability problem, treated here, were due to my my having taken a new first time IEEE Boston Section course lead by Eli Brookner entitled : *Adaptive Arrays, Sidelobe Canceler, CFAR, and Clutter Modeling.* We became alerted to many of these aspects from practical experience and from realistic simulations [37] using **TK-MIP 2.0**, a product of **TeK Associates**, available commercially for performing Kalman filter analysis and simulation (and even actual on-line implementation via use of Data Acquisition Cards, or serial port input, and/or PCI) from an easy to use Graphical User Interface (GUI). On-line tutorials and extensive application examples are also available for **TK-MIP** including an on-line self-contained professional level textbook and short course complete with lectures, tests, corresponding answers, and a guest lecturer. This software runs on 80386 or later Personal Computer (PC) processor with hardware math co-processor chip under

Microsoft Windows 95/98/NT/ME/2000 (32-bit) operating systems and does **not** presume the presence or require use of MatLab or Simulink .

TK-MIP is a software product for the PC (without needing to have or use MatLab) that we recently developed for teaching others about the theory and practice of KF simulation [23]-[34], [45]-[81] and for actually implementing KF technology and its many variations on-line for linear and nonlinear estimation and tracking. It makes modest demands on the amount of RAM a platform need have to run TK-MIP. A mere 16 Megs of RAM suffices for TK-MIP. An on-line "Help" system accompanies and is a part of TK-MIP along with informative demos and video movie-type AVI tutorials on "effective use of TK-MIP". We coddle the user as he explores running TK-MIP and its many uses.

APPENDIX A. CLT DISTINCTIONS & MODERN 40 YEAR OLD GENERALIZATIONS THAT MAY STILL BE NEW TO MOST ENGINEERS

To illustrate that mere sums of iid aren't necessarily Gaussian, consider the sums of Cauchy variates. Sums of Cauchy are always Cauchy (even an infinite number of them) and although it is bell-shaped, its tails are so fat that all its moments are infinite and so don't exist. Cauchy can arise physically as the ratio of independent Gaussians or as their arctan. CLT requires that the variance of the contributing variates be finite in order to invoke the desirable conclusion that the sum is Gaussian in distribution. Insights and pointers to the more recent CLT generalizations no longer requiring iid are available in Sec. 9.3 of [3]. In particular, the Lindberg-Feller Theorem on p. 239 of [3] doesn't require the contributing variates to be identically distributed (citing Woodroofe, 1975, p. 255) and Sec. 9.3.2 of [3] reveals the "dependent variable case" (citing Moran, p.403 & Serfling, pp. 1158-75, both in 1968).

The conclusion of the Central Limit Theorem (CLT) [and all it's variations] is that: *Sums of random variables that are independent and identically distributed (iid) with finite variance go to Gaussian in distribution (as the number of terms increases)* is still intact; however, the CLT hypotheses have been weakened more recently (40 years ago) to now allow non-identical or non-independent distributions (but not both), yielding this same classical conclusion! This weakening of CLT hypothesis to be more easily met in applications actually strengthens or more precisely widens the applicability of Irving Reed's nice result [14], as is now observed and pointed out herein.

Normally, the *estimate of the variances* for Gaussian random variables (being what occurs at fixed specified times for Gaussian random processes or stochastic Gaussian processes) involves a matrix that has a Wishardt distribution. (Please see *Analysis of Variance* by the late Prof. Scheffe). This is the path that statisticians have historically blazed rigorously. The original random process has one distribution and the moments that are sought as estimates inherit a distribution for their various statistics (in general, its not the same distribution for the various statistics but gets more challenging the higher order the moment being sought is). Please confirm by reading such classic texts on Statistics as that by Johnson and Leone [41] or by Bowker and Lieberman [42] or by David Himmelblau [36]. The Wishardt distribution can be extremely challenging to work with in posing hypothesis tests especially for multivariate situations (which is our case). Irving Reed sought to avoid the hassle (rigorously) by instead invoking the CLT to then infer that the sums of independent identically distributed variates (time invariant, statistically stationary) random variables is again going to Gaussian (in distribution). My observed extension is that even for non iid (for underlying variates with [changing nonstationay] variances) the sum still goes to Gaussian (in distribution). Hey, as engineers we already know (from [1]) that linear operations (e.g., scaled summing) on Gaussians still yields outputs that are Gaussian so everything that we have encountered so far is at least consistent. However, along ONLY the last of these lines of argument, did we have to start with a Gaussian being present. When other types of noise are present, both Reed's classical CLT and my most recent CLT update (applicable under a wider collection of circumstances) guarantee that the output of the sum is Gaussianly distributed. However for Non-gaussians, we are stymied unless we invoke the recent CLT generalization to conclude Gaussian (in distribution) outputs for the time-varying situation too and therefore presenting non-identical distributions for the individual participants in the sum.

A worry of mine is that some researchers work directly with the *complex case* (as distinguished from the *real variable* case) when analyzing these situations and sometimes appear to have a larger factor on their ending Gaussian distribution than should otherwise be the case. The reality is that the complex case is isomorphic to a 2-D exclusively real case (see Walter Rudin's *Real and Complex Analysis*, 1967). Gaussian Tables are available only for the *real case* to start with. False factors being present in distributions for the complex case could yield performance claims that are a square root of two improvement over other prior approaches. I fear that this is happening NOW! The offending engineer will remain nameless here (and perhaps will remain nameless forever).

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